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A Novel Decision-Making Framework for Robust-Reliable Aggregate Production Planning Problem

Nowatorskie ramy podejmowania decyzji zapewniające niezawodność zagregowanego problemu planowania produkcji

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Dedication

This work is dedicated to the pursuit of a better world through the lens of human factors.

This thesis is for the individuals whose deeds drive happiness, and to the belief that satisfaction, well-being, and fulfillment of humans are essential for a thriving workplace and society.

May this research contribute to a deeper understanding of what it means to create workplaces where truly flourish and empower individuals.

To those who believe that understanding people can create a safer, smarter, and more compassionate environment in both private and business life.

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Abstract

In the dynamic and complex landscape of Aggregate Production Planning (APP), achieving a balance between cost efficiency, reliability, and workforce well-being is critical for sustainable operations. This thesis develops a novel decision-making framework for robust and reliable APP, with a particular focus on integrating human factors into the optimisation process. By leveraging advanced methodologies, including Multivariate Adaptive Regression Splines (MARS), Weighted Goal Programming (WGP), and Fuzzy Programming, the research addresses the challenges of uncertainty, multi-objective optimisation, and workforce dynamics. The framework is applied to a real-world case study in the automotive industry, a sector characterised by multi-product manufacturing, fluctuating demand, and stringent just-in-time (JIT) requirements. Two key objectives - minimizing total costs and maximizing system reliability - are explored using a bi-objective Mixed-Integer Non-Linear Programming (MINLP) model. Human factors, such as learning and forgetting rates, fatigue dynamics, and workforce reliability, are systematically integrated into the model, providing a comprehensive approach to enhance both operational efficiency and employee reliability. The study's findings emphasise the critical role of workforce-related variables in achieving reliable and sustainable APP. Practical tools, such as the Matrix Questionnaires (*MQ1* and *MQ2*), are developed to evaluate and incorporate human-centric criteria into production planning. Sensitivity analyses further validate the robustness of the proposed model, offering actionable insights for managers to navigate the complexities of demand uncertainty and workforce variability. This research contributes to the growing field of sustainable production planning by bridging the gap between operational objectives and human-centric considerations. It provides a scalable and adaptable framework for the automotive industry and beyond, paving the way for future advancements in integrating human factors, advanced analytics, and sustainability into decision-making processes.

Keywords: Aggregate Production Planning, Human Factor, Multivariate Adaptive Regression Splines, Weighted Goal Programming, Fuzzy Programming, Mixed-Integer Non-Linear Programming, Robust Optimisation, Sensitivity Analysis.

Streszczenie

W dynamicznym i złożonym środowisku zagregowanego planowania produkcji (ang. aggregate production planning, APP) osiągnięcie równowagi między efektywnością kosztową, niezawodnością i dobrostanem siły roboczej ma kluczowe znaczenie dla zrównoważonych operacji. Niniejsza rozprawa opracowuje nowe ramy podejmowania decyzji dla solidnego i niezawodnego APP, ze szczególnym uwzględnieniem integracji czynników ludzkich i procesem optymalizacji. Wykorzystując zaawansowane metodologie, w tym wielowymiarowe adaptacyjne regresje splines (ang. Multivariate Adaptive Regression Splines, MARS), programowanie ważonych celów (ang. Weighted Goal Programming, WGP) i programowanie rozmyte, badania obejmują wyzwaniami niepewności, optymalizacji wielokryterialnej i dynamiki siły roboczej. Ramy te zastosowano w rzeczywistym studium przypadku w przemyśle motoryzacyjnym, sektorze charakteryzującym się produkcją wielu produktów, zmiennym popytem i rygorystycznymi wymaganiami just-in-time (JIT). Dwa kluczowe cele - minimalizacja całkowitych kosztów i maksymalizacja niezawodności systemu - są badane przy użyciu dwukryterialnego modelu mieszanego programowania całkowitoliczbowego nieliniowego (ang. Mixed-Integer Non-Linear Programming, MINLP). Czynniki ludzkie, takie jak wskaźniki uczenia i zapominania, dynamika zmęczenia i niezawodność siły roboczej, są systematycznie integrowane w modelu, zapewniając kompleksowe podejście do zwiększenia zarówno wydajności operacyjnej, jak i niezawodności pracowników. Wyniki badania podkreślają krytyczną rolę zmiennych związanych z siłą roboczą w osiąganiu niezawodnego i zrównoważonego APP. Praktyczne narzędzia, takie jak kwestionariusze Matrix (MQ1 i MQ2), są opracowywane w celu oceny i uwzględnienia kryteriów zorientowanych na człowieka w planowaniu produkcji. Analizy wrażliwości dodatkowo potwierdzają solidność proponowanego modelu, oferując menedżerom praktyczne spostrzeżenia, aby mogli określić niepewność popytu i zmienności siły roboczej. Badania te przyczyniają się do rozwijającego się obszaru zrównoważonego planowania produkcji, łącząc lukę między celami operacyjnymi a rozważaniami zorientowanymi na człowieka. Zapewniają skalowalne i adaptowalne ramy dla przemysłu motoryzacyjnego i nie tylko, torując drogę przyszłym postępom w zakresie integrowania czynników ludzkich, zaawansowanej analityki i zrównoważonego rozwoju w procesach decyzyjnych.

Słowa kluczowe: Agregowane planowanie produkcji, czynnik ludzki, wielowymiarowe regresje adaptacyjne, ważone programowanie celów, programowanie rozmyte, mieszane programowanie nieliniowe całkowite, solidna optymalizacja, analiza wrażliwości.

CHAPTER 1
Introduction

1. Introduction

1.1. Research Background and Motivation

This chapter seeks to introduce the research study by providing an overview of the dissertation's background, problem statement, objectives, hypothesis, research design, and overall organization. Setting the context for the research objectives and theory, the significance of human related variables into *Aggregate Production Planning* (*APP*) that would give significant advancements to a company by robust production planning which is optimal with respect to cost and reliability (quality). Backdrop and motivation for the study emphasize the significance of *Human Factor*s (*HFs*) in today's dynamic and unstable marketplace through questionnaires and its following analysis in *HF-APP* problem (or also called *"*HF-supported" or *"*HF-supported APP") which means developed by the contributions of *Human Factors* and demonstrate the importance of *Human Aspects* for a production planning.

Production Planning (*PP*) is one of the most attractive and essential topics in manufacturing systems, which is about efficiently planning and coordinating all manufacturing activities so that the goals of the companies or organizations are met. PP includes the main steps such as determining optimal production and inventory levels and other key production parameters that deal with demand uncertainty during a given planning period (Ramezanian et al., 2012). APP is an instrument or method to find and establish an equilibrium or approximate equilibrium between capacity and demand in PP which was first modeled by (Holt et al., 1955).

Figure 1 displays the inputs and outputs of APP, along with its objectives. This figure also shows the advantages of APP in terms of minimising and maximising cost and benefit criteria, respectively. Main strategic considerations behind APP can be outlined as follows (Mirzapour Al-E-Hashem et al., 2011):

- Changing the number/productivity of workforce (hiring, training, etc.),
- Changing the production rate (work shifts, overtime, outsourcing, etc.),
- Consolidation of periodic inventories (holding costs, peak demand periods, etc.),
- Planning and treating back-orders (balance between costs of delaying deliveries and changing production rate),
- Influencing demand (advertising, product promotion, discounts, etc.).

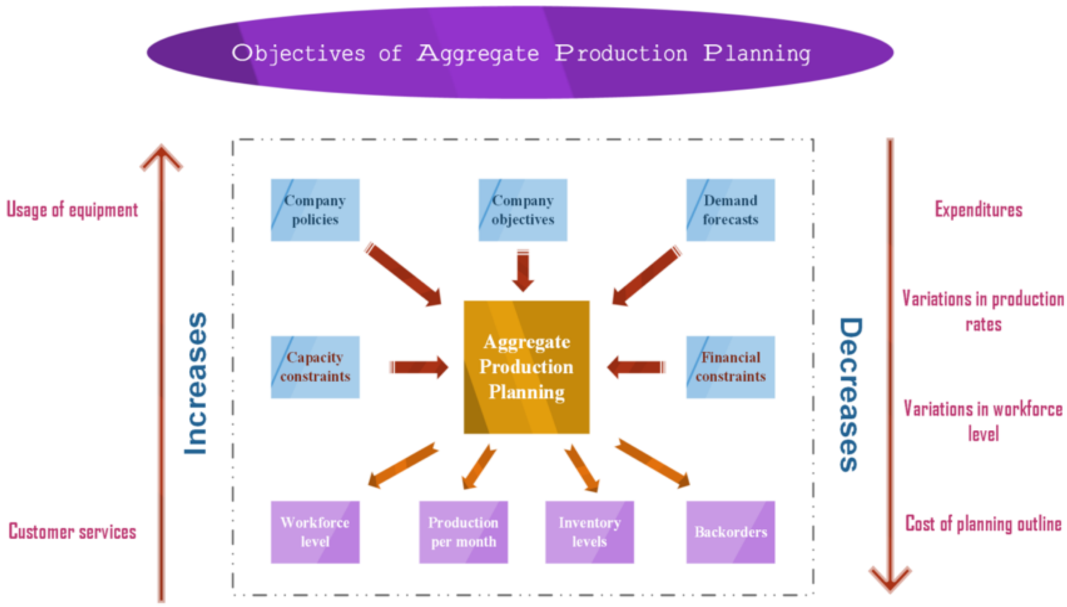


Figure 1. Objectives and inflows/outflows of APP.

Planning for Aggregate Production (AP) over the long term is still crucial and reliant on numerous variables. Although manufacturing organisations persist in utilising sophisticated and advanced technologies to construct precise planning models, human-related elements remain essential for effective and sustainable production. While the benefits of intelligent and automated systems in manufacturing process are tremendous, human labor remains central to sustainable operations in key areas due to the versatility, cognitive and motor abilities that computers cannot yet economically duplicate. However, it is not easy to effectively impose the influence of human-related factors, or the human entity, as well as technology and mechanization, into the production cycle as an efficient part of the system because humans cannot be designed or operationalised like machines due to their innate cognitive, behavioural, and emotional components. The examined literature demonstrates that, even with the recent explosion in automation and knowledge engineering tools, the development and management of resilient and adaptable industrial systems depend heavily on cognitive and social processes linked to workers. Unlike in the traditional time series, the sequence of activities in human decision-making involves several cognitive processes, including desires, beliefs, the theory of mind, and intents, the authors said. Results by Lin, Bouneffouf, and Cecchi (2022) provide empirical evidence for the critical relationship between cognitive processes and individual behaviours and corroborate earlier findings by Brough et al (2011). For this reason, in line with the theory of neural networks, Lin, Bouneffouf, and Cecchi (2022) use recurrent neural networks to predict human choices/decisions in psychological tasks.

Beratan (2007) remarked regarding the way decision-making is a cognitive process. The author pointed out that every human decision originates in a brain cell that reacts to sensory information. In terms of the cognitive underpinnings of behaviour, Beratan (2007) noted that conscious processing in brain cells combined with experimental knowledge produces decision-making that is accessible to a non-conscious mind. Cristofare (2020) examined human decision-making and behaviour using the affect-cognitive theory. According to the author's theory, adaptive affective states of the people - who are both the subjects and the objects of cognitive errors - interact with cognition and related deceit. Organisations can enhance employees' decision-making skills and steer their behaviours towards job satisfaction by concentrating on their cognitive processes and talents, according to Cristofaro (2020) findings. The association between cognitive processes and risky workplace behaviours was the main topic of Liao et al. (2014) study. The authors sought to comprehend how employees' cognitive processes affect the situations and behaviours that result in safe and unsafe workplace behaviours. The study placed a strong emphasis on how communication helps to encourage safe behaviour. Liao et al. (2014) stated that managers must continually remind staff members of the need of acting in a safe manner by setting an example. According to the findings of Liao et al. (2014), there would be less instances of cognitive failures in an organisation with a structure that promotes communication between leaders and employees because workers would be more confident in their ability to complete the task at hand, which would result in fewer risky workplace behaviours.

Failure to consider human factors in manufacturing operations could result in erroneous process designs, failing systems, and even increased employee health hazards. That is why, understanding human related factors in relation to job performance, consequently the efficiency of the organisations itself, which has a direct relation with workers' fulfilment, satisfaction and happiness, is key step towards maintaining and improving efficiency of existing systems as well as integration of new technologies with workforce to optimise manufacturing processes and remain competitive.

Effective systems can only be sustained in the long term by peacefully run, satisfied businesses that are established within happy communities, i.e., by happy people. Keeping workers in a high job satisfaction condition is essential for organisations seeking sustainable operations, as job satisfaction is connected to employee reliability and efficiency. That is why factors that have a direct effect on workforce satisfaction are some things were particularly paid attention to as well. People with the *"right skills at the right place and at the right time"* advance effective and peaceful systems, which are always in high demand in today's market locations and situations. Furthermore, long term sustainable and reliable systems can be achieved through the right use of people with their right skills, qualifications and competences. That is why, in this study, it is also another important point to analyse which skill group (among technical skills, general skills and common skills) has more impact on systems in order to achieve broader analysis when it comes to training and learning concept of the employees, even in further extensions, which education system should be applied to future employees at education places, either high schools or universities while considering the needs of market. The similar analyse has been done by Szafrański et al. (2022) and Graczyk-Kucharska et al. (2020). The authors place special emphasis on how professional competencies should be aligned with business needs in the context of the fourth industrial revolution.

However, undoubtedly, the context of satisfaction cannot be limited by being at the right place and right time. Work satisfaction is still the result of other numerous variables influencing actions taken by individuals. For example, O'Hora and Maglieri (2006) examined goal-directed behaviours and how they affect job satisfaction. This study's key finding is that goal-directed behaviours take place in organisational settings independently of reinforcement. The study found that when managers give goal statements, the statement is given in light of the employees past experiences with the company. O'Hora and Maglieri (O'Hora & Maglieri, 2006) discovered that, for the workers, operating according to the objective statement forecasts particular environmental consequences even in situations when they aren't really given. A study (2017) by Tu, Lu, and Yu examined the relationship between moral leadership and contentment at work. Tu, Lu, and Yu (2017) used structural equation modelling based on a survey of 371 workers to positively corroborate the idea that moral awareness and job satisfaction are positively impacted by ethical leadership. The authors observed that the association between ethical leadership and job happiness is mediated by the moral identity and moral awareness of the employees. In another study, Said, Abukraa, and Rose (2015) came to the conclusion that a person's personality has a role in their level of job satisfaction. The five well-known personalities, according to the authors, significantly improve work happiness. According to Steel et al. (2019), these personalities account for 13% of the variance in life happiness and 10% in work satisfaction, which is consistent with the findings of Said, Abukraa, and Rose (2015).

Haarhaus (2018) provided empirical research centred on team satisfaction within organisations. The working environment of team members had a major impact on job satisfaction, according to a pathway analysis of 415 team members and 110 groups. The author noticed that workers are badly impacted by shared affective working occurrences. Nevertheless, there was no proof that social contact might knead employment evidence into consistency. Additionally, the study verified that employees' personality attributes indirectly impact their level of job satisfaction. Medrano and Trógolo (2018) conducted an analysis that focused on an employee well-being model in Argentina and found that certain events both within and outside the workplace contribute to employees' ability to psychologically detach themselves from their work. The study by Weigelt, Gierer, and Syrek (2019), where the authors connected psychological detachment to satisfaction, further emphasises this idea of psychological detachment. As a result, organisations must concentrate on developing procedures that encourage disengagement from the office and give workers time to engage in leisure activities. The aforementioned results are consistent with the earlier study conducted by Judge and Watanabe (1993), which demonstrated a connection between job happiness and life satisfaction. The core claim of Judge and Watanabe's (1993) research is that when people are not satisfied with their lives, it is difficult for an organisation to have high employee satisfaction.

Furthermore, research has repeatedly demonstrated that companies that prioritise employee fulfillment or human resources operate more effectively (Herrbach & Mignonac, 2004). Human factors are not only limited to the physical conditions of employees, but also their social needs including cognitive elements arising from their present and even past experiences. Organisations must understand how various factors affect employee work life and create appropriate systems. For this reason, this study aims to understand human variables, and the resulting implications on work environment, workforce quality, and organisational processes.

Humans play a big role in understanding main issues and taking the right actions. However, in engineering and management science, HFs generally have long been underestimated or viewed as relatively confined to contributions on “safety” requirements or “ergonomics” whilst the other aspects are remained widely neglected (Liao et al., 2017; Xu et al., 2018). Moreover, when HFs are studied in the business world, the majority of past research relied on analyzing the product or service satisfaction of external consumers (Güçdemir & Selim, 2017; Goli et al., 2019). Only a few studies have looked at how employees (or “internal consumers”) feel about their jobs and their impact on production systems (Jung & Suh, 2019).

Nowadays, *Human Factors* are becoming more trendy and the value of human beings in the system is receiving more attention (Neumann et al., 2016). On the one hand, state-of-the-art automation and the newest technologies have already helped a lot in the modern industries and in all parts of life. On the other hand, still there are numerous quality deficiencies, and they can affect or endanger outcomes due to a lack of performance of operators, technicians and workers - of us humans. Hence, HFs can become critical, in fact “bottleneck factors” and “crystallizers”, for the reduction of errors in production, for improving quality outcomes and for enhancing the overall fulfilment towards top standards on all sides of modern production, manufacturing and consumption.

Humans have always been sources and providers of innovation, creation and problem solution. This all is affected by numerous factors belonging to the work system and brings work results. A solid understanding of the impact of the disruption of these factors is required to make strategic decisions. Therefore, the purpose of this study is to investigate the effect of the factors on efficiency of manufacturing system that are human related and general work atmosphere. Herewith, in order to get efficient work results (fulfilment demands, innovative approach, high quality work), this work draws attention to workforce related factors to increase workforce / system productivity, with further results to have sustainable and reliable systems. Then, in subsequent steps, these findings will be implemented into problem of decision making in APP.

As determined by social psychological analysis, an attitude differs from an effective reaction to a circumstance or an object (Weiss, 2002). Because of this, rather than being an emotion, job satisfaction is an assessment of one's work that is entirely dependent on the workplace. That is why, while labor fulfilment can be interpreted differently, the attitude or reflective evaluation on one’s employment and workplace in a holistic way is the main approach of this study in the case of human related manufacturing scenarios. Realising and pursuing this holistic approach is necessary to gain greater understanding of the cognitive, emotional, physical, and psychosocial aspects of human behaviour and how they relate to worker’s performances as well as organisation’s itself (Sgarbossa et al., 2020).

Therefore, in this research, by the help of two questionnaires called *MQ1* (Matrix Questionnaire 1) and *MQ2* (Matrix Questionnaire 2), the point of fulfilment is connected with many factors related to the entire work system, relationships with co-workers, human resources operations and principles, overall fulfilment of duties, quality of contribution, production level, flexibility at work, health and safety implications which were categorized as the main factors (criteria) that can affect the labor together with their sub-factors. All these main groups of factors and their sub-elements (sub-factors) are stated in the questionnaires. These sub-elements will contain the input variables for the *MARS* model, and in total, there are 50 variables (X_i's) as 2 groups of sub-elements from *MQ1* and MQ2 (15 X_{i1}'s and 35 X_{i2}'s respectively).

Herewith, the questionnaires were created by the light of all these aforementioned studies before in order to comprise all possible effective factors for the robust- reliable HF-APP. In fact, on the way to this APP model, by the results from MARS application, the most important input variables were scientifically derived for the criteria or output variables, built the scoring matrix and created new parameters / decision variables in its various realizations. More information will be given in the next chapters.

Traditionally, businesses followed set production and manufacturing procedures, operating inside preexisting frameworks and avoiding the need for creative problem-solving. Nonetheless, the modern era is marked by dynamic changes, with the highest priority being given to promoting innovation and adding value. There is a lack of analysis for humans in the systems, not as need of customers, but as one of the most important inner effect, system builders or workers. Hence, there is a need to close this gap in the literature. The motivation behind this study lies in the need to raise human effect in production system and create more efficient systems by paying attention in a comprehensive way into system's needs and human (workers) needs at the same time. As a result, in this scientific study, valuing HFs in a much broader and loftier way was aimed, and the other aforementioned goals.

1.2. Research problem, Objectives and Hypothesis

In view of global challenges, such as environmental, economic, and, more recently, human-related elements for social components, the long-term development of aggregate production planning is considered as a vital issue. In today's market, companies face complex customer requirements, uncertain demand and the need for a fast and timely delivery. This forces the companies to aim at minimizing costs for fulfilling demand while keeping the systems sustainable and reliable. That is why, in processes of production or manufacturing, *Production Planning* (*PP*) is crucial as it plays an important role in managing fluctuating consumer demand.

Effective PP also allows companies to optimize their resources. It helps balance supply and demand, meaning the right amount of product is made in order to meet customer demand. From Production Planning, *Aggregate Production Planning* (*APP*) is achieved by using *Aggregate Planning. Aggregate Planning* is some scheduling process in operations management and especially in production, as (*APP*), which contains decision making processes with respect to the quantity and the timing of manufacturing regarding a determined time period. Therefore, it is important for a company or an organisation to create a production plan which is ready to effectively fulfill these goals. Further aspects of AP may arise from environmental, social and cultural issues (Rasmi et al., 2019). AP includes the generation of monthly and quarterly plans that targets the challenging task of adjusting the production capacity under varying demand (Goli et al., 2019). For these reasons, AP should be made in medium-term time horizons within planning processes (Cheraghalikhani et al., 2019). APP is concerned with the determination of production, inventory, and workforce levels in the presence of uncertain demand and over a specific planning window up to one year (Tirkolaee et al., 2019, Mirzapour Al-E-Hashem et al., 2013). The latter commonly varies between 3 to 18 months (Noegraheni & Nuradli, 2016). Hence, APP brings businesses into strategic positions to organize their resources for achieving an effective and efficient capacity utilization (Nugraha et al., 2020).

These factors have solidified the critical importance of production planning and control, particularly within extensive supply chains. Achieving economic and efficient production demands meticulous and comprehensive planning at every stage, from sourcing raw materials from suppliers to manufacturing final products in factories and distributing them to customers. Such planning ensures the optimal utilisation of resources while minimising the total costs across the production system. AP purposes a balance between a number of controllable factors, e.g., inventory, production levels, workforce-related factors and further ones related with resources in order to adjust the production capacity to the anticipated, expected or predicted demand. Since the future continues to be uncertain, AP proposes a comprehensive, systematic and more and more holistic methodology of generating realistic business outlooks, and makes companies or organizations better prepared, fit and ready for sound decisions. That is why, in this study, the concept of the workforce aspects for a a multi-criteria production problem will be brought besides all other important factors for a production system. The difficulty here is to analyse a multi-criteria APP problem to create, for example, three products in 4 weeks of the year with market unpredictability, particularly under uncertain demand and human factor uncertainties. Herewith, the questionnaires (*MQ1* and *MQ2*) were entirely created to detail workforce-related factors. Finally, a *Multi-Objective HF-APP* model will be implemented based on two main objectives accordingly. It will be solved by the help of *Robust Optimisation*, providing us with conservative solutions regarding the uncertainty involved into the optimization program.

The goal of thesis is developing a decision-making framework which will help the practitioners to maintain sustainable and reliable systems in today's markets while considering human factors. In this study, it was aimed to bring out the relevance of human-factors and its effect for manufacturing systems. Hence, the goal of the research is creating a system unfolding a broader approach to human-related scenarios in decision-making processes to contribute to reliability and sustainability of the production systems.

Subject and objects of the study: The problem is considering a robust-reliable *multi-period and multi-objective APP problem* to produce, e.g., more than one product in four weeks of a year with market uncertainty, especially, under uncertain demand and uncertainties of human factors. Moreover, the stability of the system (which can be improved through providing the options of overtime and outsourcing) is defined as the ability to meet the customers demand, especially based on the JIT production policy.

Accordingly, two main objectives of the study are as follows:

(i) *Minimising the total cost,*
(ii) *Maximising the reliability.*

As a result, the study aims to reach Economically and Socially Sustainable & Reliable (Robust - Reliable) Aggregate Production Planning. More information will be given in the following chapters.

Research Hypothesis: The aim of this study is to demonstrate impactful human factors for a APP. The research problem was articulated through the main hypothesis of the dissertation, which says that,

H0 – The inclusion of Human Factors (or Human Resource Management operations) into APP leads to significant advancements of a company by optimised production planning, thus, for a modern economy, in terms of the 2 goals of cost minimisation, reliability maximisation.

These advancements are robust with respect to uncertainty, and they support scientific research and its contemporary implementation.

Research Questions

The research problem has mainly been considered through and in the form of the following questions:

RQ1 – What (which variables) are "Human Factors" with a possible impact on Aggregate Production Planning?

RQ2 – What input and output variables are crucial for cost-efficient and reliable decision-making?

RQ3 – What are the intercultural and multidisciplinary constraints for data collection?

RQ4 – How to process data obtained from the MQ?

RQ5 – What is the relation between HFs and reliability and cost criterion?

RQ6 – How to include and cope with uncertainty in APP?

RQ7 – What is the impact of HF on quality/efficiency of APP and its results?

RQ8 – What are the managerial implications of APP?

1.3. Research Design of the Study

In the literature, most of the research works just investigated the industrial dimension of the problem (e.g., machinery issues), but nowadays, the concepts of human related operations, reliability and sustainability of the production system have got more attention by industries to be incorporated into the *decision-making processes* such as Multi-Objective Decision Making (MODM). The study's research design comprises the methodology, data, and environments in which the current investigation will be conducted. The required data include the detailed information of the company, for example: planning horizon which can be considered weekly, monthly, seasonally and annually according to the nature of the industry, work-shifts plan, types of products to be produced in each period, levels of product quality, demand of each product for each level of quality per each period is uncertain, cost-related parameters, cost per man-hour for normal working, cost per man-hour for overtime working, inventory holding and shortage unit costs, employment and unemployment costs per man-hour, unit outsourcing production cost and advertising cost, overtime shift capacity, minimum number of the required workforce, initial inventory and shortage values (at the beginning of the time horizon), internal and external (outsourcing) production capacities, average number of failures in a year, and warehouse capacity.

This research has 2 main stages when it comes to the methodology part. In the first stage, the new data will be added, including Human Factors, collected by the *Matrix Questionnaires* together with the research partners from Iran, from a relevant automotive company *for a real-life case study*. First, some preparations will be made to the gathered data for the analysis. *Simulation* method will be used in this step in order to expand the obtained data. After that, the data mining method, *Multivariate Adaptive Regression Splines* (MARS), will be applied to provide the best values for the most important factors. Herewith, the *Score Matrix* (or *Score Table*) will be prepaed as a part of the outcomes from *MARS* models by stating the percentage values of the importance for each variable. The scores for each input variable (factor in the questionnaires) from MARS results and the weight values for each output variable (criterion in the questionnaires) have been calculated. The weight value of each factor will be calculated based on the ratio between the summed weighted values of the related criteria and the sum of the weighted values from all factors. The *scores* will be equal to the summation of the products of the importance value of each sub-factor with the weight value of each related factor (examples in equation form will be given in the next chapters). The most important factors according to those scores are determined. By this way, the most important factors will be included into the APP problem. These parameters are indeed cost- and reliability-related; they are called m_{nj}, TRC_{kn} and R_{tjnk} and will be used for the 2 objectives of the HF-APP problem.

In the second stage, the inclusion of new parameters (m_{nj}, TRC_{kn} and R_{tjnk}) into APP through cost and reliability objective functions will be applied. Additionally, *uncertainty* will be defined, e.g., as triangular fuzzy numbers. The proposed model will then be developed using *Weighted Goal Programming* (WGP) and *Fuzzy Programming*. CONOPT solver/GAMS software is used to find the solution to the final MINLP (Mixed-Integer Non-Linear Programming) model, and if needed, it can be coded by the programming languages C, C++, Python, MATLAB, etc. After the APP problem is fixed and solved, as a further step, a careful *Sensitivity Analysis* will be applied for m_{nj}, TRC_{kn} and R_{tjnk} several times in order to demonstrate the effect of each parameter.

Therefore, in this work, optimal decisions are made, and the best-possible policy is determined which can largely provide remarkable benefits. In fact, enhanced MQs, their analysis, and calibration into a detailed decision-making model will accomplish and extend the finest possible knowledge about people in the system. This will result in a well-prepared *Optimisation Problem* that will serve as the foundation for developing an optimal Aggregate Production Framework. This investigation on MQs and APPs can be used as examples in a variety of modern branches and sectors as well as in other cultures and environments.

1.4. Structure of the Dissertation

The dissertation is structured in four different chapters including *Chapter 1*: Introduction, *Chapter 2*: Literature Review, *Chapter 3*: Methodology and Main Results, and *Chapter 4*: Discussion and Conclusion.

Accordingly, the literature of APP is scrutinised in the next chapter in order to identify the research gaps and define the contributions of this work. Next, the methodology will be designed based on the outcomes of Chapter 2 as well as data collected from a real case study problem. The initial results will also be obtained to test the applicability, complexity and validity.

Finally, to discuss the findings and go further with managerial insights, a set of sensitivity analyses is performed in the last chapter followed by a conclusion and outlook to future studies, and a summaries part.

CHAPTER 2
Literature Review

2. Literature Review

2.1. *Preliminary*

APP's main objective is to determine the levels of production, inventory, and workforce requirementsrequired in order to fulfill expected consumer demand. The works of Thomas and McClain (1993), Shapiro (1993), and Silver et al. (1998) are excellent general sources on production planning. APP functions as a link between short-term scheduling and strategic planning and usually lasts three to eighteen months (Chopra & Meindl, 2021). It plays a vital role in ensuring the continuity, efficiency, and profitability of production activities, as well as the seamless operation of supply chains. However, there is a noticeable gap in the literature regarding comprehensive surveys that explore the application of diverse model structures, solution methods, and approaches to managing uncertainty. Such reviews are essential to guide researchers and practitioners in identifying new areas of study and application. One of the fundamental works in the literature was done by Holt (1955) to introduce the linear decision rule for APP, with an emphasis on workforce, inventory, and production balance. The paper offered a fundamental strategy for APP formalisation and Established APP as a tool for quantitative decision-making. In 1960, Manne demonstrated the utility of linear programming for sequential decision-making by applying it to APP. Later, Hansmann et al. (1960) highlighted the importance of including workforce and capacity restrictions into linear programming models for APPs. This advancement made APP models more useful for business applications and was the beginning of involvement of capacity constraints. Bowmann (1963) emphasised the shortcomings of conventional APP techniques and advocated for optimisation. The paper presents a few theories, concepts, and studies regarding managerial decision-making. Employment scheduling and aggregate production are the initial research issues addressed. This gives rise to the idea that management's past choices might be integrated into a framework for enhancing their current choices. Other developments for APP have happened in 1970s and lead to expansion of APP models. First pioneering work was done by Hax & Meal (1975). They combined comprehensive scheduling with APP to introduce hierarchical planning. At the next level of the hierarchy, every optimisation model places a constraint on the model. In production systems, this was one of the first instances of multi-level decision-making. Bitran and Tirupati (1993) offer an in-depth evaluation of hierarchical planning models and techniques.

Eilon (1975) conducted one of the earliest reviews on APP, analyzing five solution approaches. This research evaluated the advantages and disadvantages of methods like the Holt, Modigliani, Muth and Simon (HMMS) decision rule, management coefficients, mathematical programming, and production switching techniques for multi-period APP with predicted demand. A more systematic review was later undertaken by Nam and Logendran (1992), who examined 140 papers from 17 journals and 14 books published between 1950 and 1990, focusing on models and solution methods. Graves (1999) presented the most general formulations of relevant optimisation models and briefly explain their solution in order to give an overview of the field. Decades later, Cheraghalikhani et al. (2019) and Jamalnia et al. (2019) revisited the topic with comprehensive literature reviews, filling the gap with updated insights after 27 years. These research papers are good guidelines to follow the historical advancements of APP models and solution algorithms.

Another significant advance for manufacturing systems is the consideration of uncertainty. The literature on production planning under uncertainty is reviewed in part in the work of Mula et al. (2006). The goal of the study is to give production management academics a foundational understanding of uncertainty modelling in production planning challenges. The 87 citations in the literature review were put together including the years 1983 through 2004. The study of Fahimnia et al. (2015) offers a comprehensive review of supply chain risk management quantitative and analytical models (i.e., mathematical, optimisation, and simulation modelling approaches). It investigated supply chain risks and environmental factors in APP models.

This timeline shows how APP has developed over time. These works address issues including sustainability, uncertainty, optimisation, the incorporation of contemporary technologies into production planning frameworks, and they collectively constitute the major contributions to the literature review on APP.

In the following, theoretical background as well as the most relevant and important studies are reviewed in terms of mathematical models and solutions methods along with stochastic, statistical and AI methods used to address the problem.

2.2. Theoretical Background

Since the future continues to be uncertain, AP proposes a comprehensive, systematic and more and more holistic methodology of generating realistic business outlooks, and makes companies or organisations better prepared, fit and ready for sound decisions. The core goals of AP are these:

I. *Reduction of investment in inventory*,
II. *Maximisation of contribution to the profit*,
III. *Minimisation of work-force level, and of its changes*,
IV. *Maximisation in utilising production facilities and equipment*.

APP not just optimally determines the production levels and the portfolio or mix of resource inputs, but rather also chooses most cost-effective means to satisfy given, identified or assessed requirements, including demand, while obeying given constraints, especially capacities (Yu et al., 2022). APP knows 3 basic strategies, namely:

I. *Level strategy*,
II. *Chase strategy*, and
III. *Mixed* (or *hybrid*) *strategy*.

Through processes of a separate or a "switching" mixed use of these 3 strategies, a company or organisation continues to adjust and can even achieve profits.

- **Level Production Strategy**

Here, the production rate is preserved at a steady level independent of the fluctuations in the demand. During times of low demand, unused or unsold (excess) production is kept in the stores. This inventory is later utilized to meet increased demand, e.g., in periods of demand peaks. The aim is to keep the production consistent, with a stable use of resources and workforce. Level production strategy allows for a load or burden better predictable for the employees, but possibly to the expense of the company or organization. For businesses that value price control and operational stability over adaptability to changes in demand, the Level Production Strategy is adequate. It functions best when inventory costs are controlled and demand fluctuations are predictable. Stevenson (2011) investigated level production techniques, emphasising how they may be used in situations where production systems are stable and demand is predictable.

- **Chase Demand Strategy**

Here, the production rate is adjusted to ideally exactly fulfill the demand. If the demand goes up, the production goes up accordingly. If the demand goes down, the production goes down accordingly. Chase demand strategy commonly uses hiring and firing, employing and laying off temporary or contract workers, adjustment of shifts, overtime and even outsourcing to match the fluctuation of demand. Hansmann and Hess (1960) made the initial example of this type of models. This approach is in contrast to the Level Production Strategy, which leverages inventory to absorb fluctuations in demand while maintaining a constant production rate. Nam et al. (1992) examined APP strategies, including chasing demand, and discussed the advantages and limitations of using them for production scheduling. Other contributions have been made by Heizer et al. (2020) and Ghazvini et al. (2013). These works examined the function of the chase demand approach in dynamic production systems for cost reduction and uncertainty management.

- **Mixed or Hybrid Strategy**

Here, chase demand and the level production strategies are getting combined. A company could on the one hand manufacture at a consistent or even constant rate, while on the other hand it can make adjustments using overtime, subcontracting and even outsourcing, if the demand goes beyond the regular labor capacity. This approach aims to maintain a balance or trade-off between managerial flexibility (especially with respect to demand) and resource stability.

The preference, choice or selection of an APP strategy is also influenced by factors like, e.g., the company's - or organisation's - criteria, goals or objectives in view of customer service, inventory and workforce, but also the cost of resources, the level of demand variability, and essential characteristics of the industry. Managers usually choose and determine a combination, mix or portfolio of these strategies to effectively cope with fluctuating demand, and at the same time they minimize costs and optimise resource utilization. The latter optimisation tasks can be done through minimisation or maximisation, according to the model developed or preferred by them.

Through a case study in the beverage business, the authors present a decision model that combines mixed chase and level techniques for APP, especially in unpredictable circumstances (Jamalnia & Yang, 2017). Anand et al. (2017) emphasised the use of a combination of strategies to maintain a balance between workforce and inventory levels.

2.2.1. General Typology of APP Models

Cheraghalikhania et al. (2019) grouped APP models into the classes of

1. *"deterministic"* vs.
2. *"uncertain models"*,

respectively. In their deterministic models, the parameters typically represent costs such as the backorder, inventory, labor, production and subcontracting costs, market demand or production rate. These parameters are all considered as known (or "certain") at the beginning of the planning. In contrast, their uncertain counterpart models are comprised of

- *"fuzzy models"* vs.
- *"stochastic models"*.

- **Deterministic**

Deterministic Applied Probability and Programming (APP) models are mathematical models in which neither the parameters nor the results are subject to randomness or uncertainty. Results from these models are predictable and repeatable since all input variables and relationships are accurately known and stay constant. These models are frequently employed in systems analysis, optimisation, and decision-making when uncertainty can be disregarded or the environment is stable. As the key parameters of deterministic model, all parameters are known and they are deterministic, which means same input will always give the same results.

Deterministic models have had a major impact on a plenty of scientific and engineering fields. George Dantzig's 1947 introduction of the simple approach, which allowed for effective solutions to optimisation problems with linear constraints, was a major contribution to the formalisation of linear programming in the 20th century. In the middle of 1990s, in game theory, network optimisation, and integer programming, deterministic models were extended. Agrawal et al. (2010) mentioned that the concept of deterministic models was expanded upon by Joseph-Louis Lagrange and Leonhard Euler to optimise functionals, resulting in the Euler-Lagrange equation. The findings were foundation for locating surfaces, curves, or routes that maximise a specific functional, like the quickest route between two places. In the end of 1990s, by introducing piecewise-deterministic Markov processes, Davis et al. (1984) helped to bridge the gap between stochastic and deterministic approaches. Richard Bellman formalised deterministic and stochastic dynamic programming for sequential decision-making in his works Bellman et al. (1952, 1954, 1970). By 2000s, real-world issues in many areas like finance, healthcare, and logistics were resolved by large-scale deterministic models made possible by advanced algorithms and processing capacity. In order to cope with uncertainty in real-world applications, deterministic models are enriched with probabilistic techniques and it brings the concept of hybrid models into literature. Van Der Schaft et al. (2007) provided a comprehensive analysis of hybrid systems that combine stochastic transitions and deterministic control. Nicolescu (2018) drew attention to the application of hybrid models, which include probabilistic and deterministic components, in embedded system design. Shi and Xu (2016) integrated stochastic analysis and deterministic restrictions for reliability engineering. Hespanha et. al. (2004) and Khansa (2012) have applied hybrid modeling applications in different fields like mobile commerce and innovation, and network communication.

Since the advent of the Industrial Revolution, and even more so since the triumph of computers, the Internet and AI, a useful APP can no longer deny the existence of *"uncertainty"*. Therefore, every modern and every future APP will always assume an *"uncertain model"*.

If in the APP of this thesis the demand was not assumed to be uncertain, would not be considered the data points in its underlying dataset as random variables, and in particular if the naturally uncertain *"human factors"* were not emphasized and machine failures were simply ignored, then the APP could be called a *"deterministic model"* rather than an "uncertain model". Then, the methods of more "classic", deterministic optimization would be used. Unfortunately, their results would not be realistic, not at all useful, but misleading. Sadly, this has sometimes been overlooked in operational practice without scientific advice in many countries and companies.

This thesis with its pronounced *"hybrid model"* has aimed to provide a certain remedy here.

- **Uncertainty**

According to Galbraith (1973), uncertainty is the discrepancy between the amount of knowledge previously known and the amount needed to complete a task. Numerous types of uncertainty impact industrial processes in the real world. They are divided into two categories by Ho (1989): *(i)* system uncertainty and *(ii)* environmental uncertainty. Demand and supply uncertainties are examples of environmental uncertainties that extend beyond the production process. System uncertainty is associated with production process uncertainties, including, but not limited to, production lead time uncertainty, operating yield uncertainty, quality uncertainty, production system failure, and product structure variations. Formalising the uncertainty in production systems has been the subject of numerous studies and applications over the years (Yano and Lee, 1995; Sethi et al., 2002). There is an abundant of research on production planning in the context of uncertainty. Various strategies have been put out to deal with various types of uncertainty.

The idea of uncertainty in Aggregate Production Planning (APP) has evolved throughout time, reflecting the necessity to handle unpredictability in variables such as demand, costs, production rates and supply. The development of uncertainty into APP can be linked to the growing complexity of production systems and the understanding that old deterministic models were insufficient for dealing with real life unpredictability. Since the 1970s, there has been a substantial amount of study on incorporating uncertainty into Aggregate Production Planning

(APP). As one of the first noticeable contributions, Holt, Modigliani, Muth and Simon (1956) established the foundation for the eventual inclusion of uncertainty in production planning by developing the HMMS model despite being deterministic in nature. The study (Buxey, 1979) explored the influence of uncertain demand in production planning and how it affects collective production strategies. Later, in order to address the difficulties of coordinating production across several items with demand fluctuations, Newman and Yano (1979) developed multi-item production planning with uncertainty. In the 1980s and 1990s, the incorporation of uncertainty and human aspects into APP gained prominent position with the use of fuzzy logic and behavioural models. Wang and Fang (1988) and Zimmermann (1991) were among the first to incorporate these factors into production planning systems. In 2000s and beyond, behavioural economics and cognitive psychology became more popular in research regarding, with an emphasis on decision-making biases and heuristics. Neuro-fuzzy systems and hybrid models combine computational and human-centric methodologies. The APP model presented by Ning et al. (2013) involves uncertainty theory in a way that variables such as, e.g., market demand, production and subcontracting costs, are included in the model where they appear as uncertain factors. Their criterion, goal or objective is to maximize the "belief degree" of achieving a profit which exceeds a predetermined (threshold) profit over an entire planning period. These preliminary research paved the path for more advanced models that take into consideration various sorts of uncertainty in production planning. Jamalnia and Yang's (2019) literature survey gives an in-depth examination of how it evolved of APP under uncertainty.

- **Fuzziness**

Fuzzy set theory can be used to study uncertainty caused or initialized by unknown. In 1965, Zadeh originally presented the idea of a fuzzy set theory by selecting a best decision from a set of finitely many decisions designed according to the fuzzy set. "Fuzzy set theory" is practical value when it comes to address ambivalent or ambiguous, blurred or unclear conditions which frequently occur in models of APP.

Fuzziness is a common feature in fields in which judgment, evaluation and decision-making by us humans are very important (Bellman & Zadeh, 1970). There have been important contributions to the theory of fuzzy numbers and their applications in early times (Nguyen, 1978; Kaufmann & Gupta, 1991). In theory, the best decision is the one that satisfies multiple objectives with different fuzzy-term goals. The preferences of these objectives are then aggregated using fuzzy numbers. Fuzzy set theory has been extensively reviewed and applied by Zimmermann (2011). Core applications of fuzzy numbers are mainly seen in data analysis and in decision-making processes. Lee (1990) studied fuzzy overall PP of a single product type under fuzzy objectives, fuzzy work levels and fuzzy needs along different time periods. Li et al. (2010) developed the idea of triangular intuitionistic fuzzy numbers. In addition, Kuchta (2010) proposed fuzzy numbers for a realistic project planning and project control. Zhang et al. (2014) proposed a novel approach to compare fuzzy integers based on a preference of a fuzzy probability. Total market demand for a product at any point in time is unpredictable in actual PP systems. While the production capacity can be modelled by fuzzy numbers with tolerance, the demands can be reflected by a random number with probability distribution, as a fuzzy triangular number or as an interval number. Tang et al. (2003) focused on a fuzzy formulation and simulation approach to APP problems involving multiple products with fuzzy demands and fuzzy capacities under the financial constraints in manufacturing environments.

Furthermore, Robust optimization (RO) is a powerful methodology to mathematically formulate uncertainties (Khalilpourazari and Hashemi, 2023). The development of robust optimisation in APP is essentially a reaction to the increasing demand for computationally efficient, resilient, and uncertainty-adaptable solutions. Theoretical developments and real-world applications continue to push its advancement. The foundation of RO is based on the formulation of uncertainty sets that characterise the type or range of input parameter uncertainty. Generally speaking, RO aims at 2 key factors: *Feasibility robustness* ensures that the solution provided by the robust model remains feasible for almost all realizations of the uncertainties, while *optimality robustness* confirms that the robust model solution remains nearly optimal for all realizations of uncertain parameters. As a result, RO provides with a solution which is *resilient* to changes of variations in uncertain parameter values (Fazli-Khalaf et al. 2019).

Robust optimisation makes the assumption that the unknown parameters are part of a constrained set rather than depending on probabilities. Early concepts of worst-case optimisation started to appear, especially in decision-making and control theory. In 1970s - 1980s, the control theory literature adopted a worst-case philosophy out of concerns for stability, leading to the development of robust control methods. One of the pioneering work was done by Kwakernaak (1993) during this time. The paper attracted a lot of attention promptly since it addressed some of the fundamental issues of "classical" control theory. Soon after, it was recognised that the method allowed for much more immediate control of robustness than other optimisation techniques, it was expanded to other generic situations. Robust optimisation, or RO, was widely developed in the 1990s by Aharon Ben-Tal and Arkadi Nemirovski to solve optimisation problems with deterministic variability in parameters. After all this advancement, robust optimisation was extended to inventory management, discrete optimisation, and other operational research areas by Bertsimas et al. (2003, 2006).

The integration of new algorithms and tools into optimisation software, such as Gurobi, CPLEX, and MOSEK, has improved robust optimisation as computer capacity has increased. Robust optimization has benefited from these tools and has generated better results. In the study Gearhart et al. (2013), authors used a set of linear programming test problems to evaluate four open-source solvers, and the outcomes have been compared with the industry standard IBM ILOG CPLEX Optimiser (CPLEX). The considered solvers were COIN-OR Linear Programming (CLP), GNU Linear Programming Kit (GLPK), lp_solve, and Modular In-core Nonlinear Optimisation System (MINOS). This study's main objective was to examine open-source solvers, but it also shows how beneficial it is to use a commercial tool like CPLEX because none of the open-source solvers could equal its performance. But this study also shown that in situations where a tool like CPLEX is not an option, there are other competent open-source solvers accessible. Other similar studies done by Mittelmann (2002) and Netlib (2012), and those studies were considered as the most informative sources for the comparison of linear solvers for different problems.

A new development in APP is adaptive robust optimisation, which allows decisions to be taken progressively while accounting for fluctuating uncertainty. This combination has greatly expanded the application of robust optimisation in dynamic and multi-stage decision-making situations. In this study, *Robustification* is applied based on a fuzzy-uncertainty framework for coping with demand uncertainty.

- **Stochasticity**

APP uses stochastic models to address the inherent variability and uncertainty in fundamental components such as:

- *Demand:* Customer demand fluctuates frequently, prompting probabilistic modelling to account for sudden increases or decreases.
- *Supply chain disruptions:* Fluctuations in raw material availability and supply chain delays.
- *Production processes:* Uncertainty in levels of output or equipment failure.
- *Costs:* Varying production or holding costs due to changes in the market.

In the 1980s, researchers began to use increasingly powerful probabilistic techniques, such as Markov Chains and Dynamic Programming, to handle decision-making under uncertainty. King and Love (1980) introduced stochastic inventory models in APP that account for demand distributions and service-level constraints. Later, Fuzzy Logic was included to deal with unpredictability in demand and other inputs, while robust optimisation addressed the worst-case scenarios. For a study on PP in stochastic demand environments, one of the early contributions was done by Feiring (1991). The PP in his study tries to optimise production levels, inventories, and workforce while balancing costs under demand fluctuations, requiring stochastic demand modelling. Through coming decades, hybrid models integrate stochastic programming, machine learning, and simulation (Birge & Louveaux, 2001; Almeida & Duarte, 2011; Shapiro et al., 2014).

Stochastic calculations and tools, methods and models are usually based on randomness in data of all origins and types, in human considerations or actions, as well as on the principles of probability theory and stochastic analysis. Their designs serve to address, evaluate, and ultimately overcome uncertainties that can be represented by probability distributions. They are also essential foundations of stochastic optimisation, stochastic programming and stochastic optimal control (Uğurlu, 2017, 2018).

Rossi et al. (2008) presented stochastic constraint programming, which models combinatorial decision problems with uncertainty by combining stochastic and decision variables. In this work, Rossi stated that general-purpose constraint solver is used to solve the constraints that the user specifies. This created the fundamental concept of constraint programming. Manandhar (2009) introduced a scenario-based method to stochastic constraint programming, which enables the use of scenario trees to model choice problems under uncertainty. Additionally, Savage (2006) covered the field of probability management in his article. It helps people make better decisions when faced with uncertainty by organising uncertainties into logical data models.

2.2.2. Human Paradigm for APP

Aggregate Production Planning (APP) has evolved to incorporate human factors to handle workforce-related complications guaranteeing that production plans are not only profitable but also sustainable and employee-friendly. Models that take consideration of employees' preferences, skills, and well-being have replaced solely quantitative and deterministic techniques, as seen by the inclusion of human aspects. Although Holt, Modigliani, and Simon's (1955) and other early models of APP included workforce modifications, they only considered labour as a numerical variable, ignoring human preferences and capacities.

The human factor was gradually included into Aggregate Production Planning (APP), as production systems and planning methodologies acknowledged the importance of worker concerns in addition to traditional optimisation aims such as cost minimisation and resource efficiency. Early APP models were essentially mathematical, focussing on cost optimisation under restrictions like as capacity for production and level of inventory. The earliest sost-minimization models, like Dzielinski and Gomory (1965) and Lasdon and Terjung (1971), were used to simulate changes like hiring, firing, and overtime without taking into account factors like employee fatigue training, or morale. Human involvement has been minimal to non-existent. Workers were considered as uniform, fixed resources, with no regard for their individuality, interests, or well-being. As APP models became more advanced, labor-related limitations (such as availability, skill levels, and labour prices) have been integrated into mathematical models. Human aspects were introduced in an indirect way, with a focus on staff availability and overtime limitations as restraints rather than human well-being or mobility or adaptability (Holt et. al,1960). The value of labour adaptability has begun to emerge, especially in industries with unpredictable demand. It was observed by Buffa & Sarin that focussing on cross-training workers to undertake numerous jobs, allowing for dynamic allocation of workforce during peak times (Buffa & Sarin, 1987).

In the 1990s, human factor effect has started to be considered in Ergonomics and Fatigue concepts. The emerging discipline of ergonomics has highlighted the consequences of physical and mental exhaustion on productivity of employees and error rates. APP models started to consider worker fatigue, break times, and ergonomic limitations in order to promote staff safety and productivity over the long run. One of the most important studies was done by Krajewski & Ritzman (1996) introducing shift scheduling models optimise rest breaks while minimising repetitive stress injuries.

By the 2000s, human paradigms had shifted, and workforce choices had become more integrated. The human aspect has shifted from limitations to dynamic components, with planning models taking into consideration workforce needs, satisfaction, and different skills. Models focused to strike a balance between worker preferences and organisational aims, which enhanced morale and minimising turnover (Bechtold & Jacobs, 1990). Another study done by

Bonekamp & Sure (2015) underlined how important it is to upskill staff members so they can work with cutting-edge technology. It was also investigated how to include decision-making by humans into APP decision-support systems (DSS) (Balakrishnan, & Cheng, 2007). Human innovation, adaptability, and ethical concerns are now taken into account by real-time systems. Humans are now seen as decision-making partners rather than only as workers (Tortorella et. al, 2019). In today's dynamic world, when interruptions occur, APP models use human flexibility and problem-solving abilities to keep things running. To illustrate, during COVD-19, when automated systems failed, human interference improved plans and solved the problems. The study (Ivanov & Dolgui, 2020) emphasised how important human decision-making is in reducing risks during unexpected situations. By the inclusion of Industry 4.0, even though there was advanced automation and data analytics methods, human-centric design placed a strong emphasis on the collaboration between machines and people.

Performance in smart manufacturing systems can be improved and biassed decisions can be reduced by prioritising human-centric approaches in production planning (Ahram et al., 2022; PlanetTogether, n.d.). For an APP to function well in the face of uncertainty, human paradigms and dependability must be taken into account. According to Gopalakrishnan et al. (2021), research emphasises the significance of human factors, reliability, and uncertainty in APP. Since early modifications to the work and product are easier to adopt and less expensive, it is crucial to incorporate human considerations early in the production planning process (Jabrouni et al., 2022). The latest advancements in production processes, according to Monostori and Kovács (2023), demonstrate a collaborative and human-centered approach to manufacturing, indicating a paradigm shift towards Industry 5.0.

2.2.3. Industry 4.0 and APP

Aggregate Production Planning (APP) is greatly impacted by Industry 4.0, which makes use of cutting-edge technology and promotes a better interaction between technological systems and human resources. By increasing productivity, adaptability, and decision-making, Industry 4.0's technologies (for example, IoT sensors and big data analytics) greatly contribute to APP processes. Big data analytics integration makes it possible to process enormous volumes of production data in real time, which improves planning for resources and forecasting demand (Luo, Thevenin, & Dolgui, 2022). This increases APP's flexibility in altering plans in response to dynamic changes in demand or supply (Wang et. al, 2016; Flexis AG, n.d.). Luo mentioned that planners can simulate many situations and choose the best course of action without affecting with real-time operations thanks to technologies like digital twins, which enable the creation of virtual models of industrial processes (Luo et al., 2022). Authors claim that, in Industry 4.0, machine learning algorithms foresee demand trends and machine breakdowns, allowing APP to account for uncertainty and optimise production plans (Lee, Bagheri & Kao, 2015; Ivanov et al, 2023). Furthermore, when artificial intelligence and machine learning are combined, intelligent decision-support systems can be created that can automatically modify production schedules in response to real-time data and unplanned interruptions (Wang & Zhang, 2021). Real-time data exchange among suppliers, manufacturers, and distributors improves synchronisation (Ivanov & Dolgui, 2020).

Data from market research in industrial sectors implies that in 2015 just 10% of the operational tasks were automated, whereas a projection sees this figure with 25% in 2025 (Xu et al., 2018). Such new technologies are driving the ideas of "Industry 4.0" technology as a new version of Industries 1.0, 2.0 and 3.0 (Lasi et al., 2014). Krishnan et al. (2024) stated that Industry 4.0 technology has gained recognition in the field of APP, including scheduling, especially in the industrialized world.

Bonekamp and Sure (2015) examined the ways in which the transition to smart manufacturing alters job responsibilities, highlighting the growing need for workers with technical skills and flexibility. Hirsch-Kreinsen (2016) supported Bonekamp and Sure by investigating the effects of Industry 4.0 on workforce dynamics, including skill needs, job redesign, and the proportion of human to machine tasks. In 2017, Schwab highlighted the need for reskilling, job displacement, and skill mismatches as well as the broader societal and worker effects of Industry 4.0. In another study, Sima (2020) discussed regarding the consequences of Industry 4.0 on workforce transformation, particularly the manufacturing sectors' move towards digital skills and the need of continuous education. Hecklau et al. (2016) suggested a competency model for Industry 4.0 workers that identifies key skills like problem-solving, process comprehension, and technical proficiency. This contribution provides a strategic approach to employees qualification. Frey and Osborne (2017) examined how susceptible jobs are to automation under Industry 4.0, focussing on the transition from manual to technology-driven jobs. In this paper, authors address the question: To what extent are occupations susceptible to automation? By the help of their novel approach, authors first determine the probability of computerisation for 702 particular occupations in order to evaluate this. Pfeiffer (2016) addressed issues with human work in Industry 4.0 and proposed a hybrid approach in which machines and humans cooperation rather than compete. In this article, the main conclusions of qualitative analysis on assembly work are summarised.

2.2.4. Industry 5.0 and APP

Building on the achievements of Industry 4.0, Industry 5.0 makes ensuring that technology improvements are in line with societal needs and human values. Industry 5.0 is driven by people and emphasises sustainability, personalisation, and teamwork, whereas Industry 4.0 is driven by technology focusing on optimizing the systems for efficiency and increasing the speed (Kagermann et al., 2013). The goal of Industry 5.0 is to humanise industrial systems by utilising technology to empower and work alongside employees rather than to replace them. By putting human aspects first, it seeks to build workplaces that are safer, smarter, and more satisfying while addressing ethical and societal issues and encouraging innovation. The primary objective of Industry 4.0 is process automation, which often eliminates the need for humans in drepetitive tasks. Although humans manage and maintain automated systems, they are not closely involved in the manufacturing cycle. However, with the concept of Industry 5.0, humans collaborate with machines, particularly AI and robots (like cobots), to improve innovation, creativity, and personalisation (Demartini, 2022), which means Industry 5.0 integrates sustainability while incorporating human creativity (Xu et al., 2021). Collaborative robots (cobots) and AI-assisted equipment can work alongside humans, synchronising labour capacity planning in APP with real-time operational requirements (Tortorella et. al, 2019). This has resulted in rather of displacing inputs by human, technology enhances human strengths. Additionally, although Industry 4.0 uses automation to cut waste, sustainability has not been a top priority (Rüßmann et al., 2015), whereas social and environmental responsibility are now central to industry 5.0 (Carayannis & Morawska-Jancelewicz, 2022).

Furthermore, most of the existing systems focus on human centric values at limited levels and ethical and social considerations are not concerned at their goals. However, with the advancements of industry 5.0, human dignity, ethics and inclusion has started being emphasized in the workplaces. This has brought up the need of better understanding human beings for the sustainable and reliable systems in today's business world. In this PhD study, it has been aimed to rise the importance of understanding of human aspects for manufacturing systems in order to achieve efficient systems. Table 1 shows the role of human beings in the technological developments as per Industry 4.0 and Industry 5.0.

Table 1. The role of human beings in Industry 4.0 and Industry 5.0.

Aspects	Industry 4.0	Industry 5.0
Focus	Efficiency, automation, digital transformation	Sustainability, personalization, human machine collaboration
Human involvement	Minimal	Active use of technology; improved decision-making and creativity
Personalization	Limited	High
Sustainability	Efficiency-driven, with resource optimisation leading to sustainability.	Centred around system sustainability and ethical issues
Technology aspect	Autonomous systems	Collaborative and augmentative
Ethics	Limited emphasis	High emphasis

As it can be seen from Table 1, the aspects for technological developments in systems mostly focus on human and human related aspects stay as main points for improving systems. Even though plenty of advancements in different fields occur every day, systems cannot by-pass humans, even more the role of human aspects are getting more and more indispensable.

2.3. Relevant Studies: Mathematical Models and Solution Methods

Over the past three decades, researchers have highlighted the importance of APP in the industry by proposing various extensions to its framework. Jamalnia et al. (2019) carried out an extensive review, examining multiple dimensions of APP in the context of uncertainty. Their study provided an in-depth analysis of significant APP research conducted up to 2018.

Türkay et al. (2016) proposed a mathematical APP model that incorporates the three key pillars of sustainability commonly discussed in the literature: environmental, social, and economic criteria. They validated their model by applying it to a real-world case study. Similarly, Rasmi et al. (2019) introduced a multi-objective APP model that expands on traditional frameworks by including not only economic, social, and environmental dimensions but also cultural aspects. Unlike conventional APP models that focus solely on economic factors, their model demonstrated its effectiveness through an example solved using an exact solution method within a multi-objective mixed-integer linear programming (MOMILP) framework. The study found many non-dominated points in the objective function space and analyzed the trade-offs, and also provided an extensive examination of the non-dominated points of sustainable APP problems. Hahn and Brandenburg (2018) introduced a hierarchical decision support approach that combines a deterministic linear programming (LP) model with a stochastic aggregate queuing network model. This method aims to enhance decision-making for aggregate production planning (APP) in the chemical process industry, which involves intricate manufacturing operations. Their work emphasized carbon emissions, sustainable operational planning, and campaign planning aligned with operational processes. They demonstrated the effectiveness of their approach through a case study from the chemical industry.

Given the critical role of energy in production, several studies have integrated energy considerations into production planning. Modarres and Izadpanahi (2016) proposed an APP model that simultaneously addresses energy planning, demand, and production capacity through three objective functions aimed at minimising operating costs, energy costs, and carbon emissions. To manage uncertainties in input data, such as demand and cost parameters, they employed a robust optimisation (RO) approach to produce resilient solutions that surpass deterministic models under uncertain conditions. Their approach was validated using a real-life case study. Similarly, Chaturvedi (2017) focused on energy-efficient production planning by incorporating capital cost considerations. They introduced an insight-driven graphical method for multi-facility APP, designed to accommodate capital expenses while minimising energy consumption in production facilities. The effectiveness of the method was demonstrated through several illustrative examples, showing a significant potential for energy savings.

Given the uncertainties and fluctuations in production parameters in real-world industrial settings, it is essential to account for uncertainty in APP models. Mirzapour Al-e Hashem et al. (2011) addressed this challenge by proposing a stochastic programming approach for a multi-period, multi-product, and multi-site APP problem under demand uncertainty. They developed a mixed-integer nonlinear programming (MINLP) model within the context of a green supply chain, incorporating indicators such as greenhouse gas (GHG) emissions and waste management. The validity of their model was demonstrated through a practical example. Khalili-Damghani and Shahrokh (2014) employed a multi-objective mixed-integer linear programming (MOMILP) model for a multi-product, multi-period APP problem. To address the model’s multiple objectives, they utilized a fuzzy goal programming (GP) approach and applied their model to a real-world industrial case study. Similarly, Gholamian et al. (2016) proposed a multi-objective MINLP (MOMINLP) model with conflicting objective functions for APP under demand uncertainty in a supply chain context. They used a fuzzy multi-objective optimisation method to effectively handle the model's multiple objectives.

Entezaminia et al. (2017) employed the RO approach to address a multi-site, multi-period, multi-product APP problem. Their model incorporated candidate collection and recycling facilities and was validated through a green supply chain case study. Similarly, Goli et al. (2019) applied the RO method to a mixed-integer linear programming (MILP) model designed to handle demand uncertainty. To address the multi-objective nature of the model, they used a GP approach alongside meta-heuristic methods, such as the multi-objective invasive weed optimization (MOIWO) algorithm and the non-dominated sorting genetic algorithm II (NSGA-II), to solve the problem. Tirkolaee et al. (2019) developed a fuzzy multi-objective MILP model for a novel multi-period APP problem under seasonal demand. The model aimed to minimise total costs while maximising customer satisfaction levels. Its validity was demonstrated using the weighted goal programming (WGP) technique with the CPLEX solver. Meanwhile, Darvishi et al. (2020) explored supplier selection, logistics decisions, and multi-site APP in the textile industry. They proposed a mixed-integer nonlinear programming (MINLP) model within a hybrid fuzzy-stochastic framework and utilised a robust two-stage stochastic programming technique to define optimal policies under complex uncertainty. Djordjevic et al. (2019) employed a fuzzy linear programming model to formulate the APP problem in an automotive industry emphasising production and inventory operation times as performance indicators. By incorporating uncertainties in demand, production, and logistics using fuzzy sets derived from historical data, the model effectively addressed real-world variability. Results from experiments with industry data demonstrated improved operational efficiency and reduced processing times, highlighting its practical applicability. Jang and Do Chung (2020) suggested a robust optimisation method to model the APP problem with implementation error under workforce hiring and layoff uncertainties using a robust optimisation approach. To avoid the conservatism of traditional robust models, a bi-level particle swarm optimisation (PSO) framework was developed, ensuring feasible and robust solutions. Experimental results highlighted its superiority over deterministic and conventional robust models in minimising average and worst-case costs while reducing product shortages under high uncertainty.

Liu and Yang (2021) developed a bi-objective mathematical model to address the APP problem and used a local search-based genetic algorithm (GA) for its solution. The objectives were to minimize total production costs and workforce fluctuations simultaneously. Attia et al. (2022) highlighted the role of organizational learning in APP, focusing on minimising total costs. They formulated the problem using a mixed-integer linear programming (MILP) model and demonstrated its applicability through a real-world case study. Aydin and Tirkolaee (2022) provided a comprehensive and systematic literature review on APP. They categorised various studies based on criteria such as model structure, solution methods, and approaches to managing uncertainty, with particular emphasis on sustainable development and the integration of digital technologies. Tirkolaee et al. (2023) offered a sustainable and robust APP model addressing supplier resilience, workforce productivity, and outsourcing under uncertainty.

Using a hybrid MADM-MODM framework combining BWM-WASPAS-Neutrosophic and multi-objective MILP, the approach optimised cost, environmental impact, and supplier selection at the same time. Validated through a real case study, the method demonstrated high efficiency and sensitivity to supplier-related uncertainties, providing valuable managerial insights. Recently, Gómez-Rocha et al. (2024) proposed an enhanced APP model for multi-product, multi-period production under demand uncertainty, incorporating the option to rent extra warehouse space. Using real industry data, the model outperformed traditional approaches by improving production costs and demand satisfaction. It provided practical insights for manufacturers to optimise inventory space while ensuring customer service levels.

2.4. Relevant Studies: Statistical and ML/AI tools

The literature reveals a growing trend in utilizing Machine Learning (ML) applications to address production planning problems and enhance manufacturing systems. ML methods have been applied for tasks such as predicting production variations, processing data, determining capacity requirements, and benchmarking against proposed models. For instance, Mori and Mahalec (2015) utilized Artificial Neural Networks (ANNs) and Support Vector Machines (SVMs), both supervised ML techniques, to evaluate the performance of their Bayesian network models. These models were designed to estimate production loads and scheduling times in steel plate manufacturing. Based on real-world data, their findings demonstrated that the models effectively predicted probability distributions for uncertain production scenarios.

Garre et al. (2020) applied ML algorithms to predict production deviations in the food industry. Their research focused on addressing uncertainties in production planning and reducing environmental impact by managing food waste. They concluded that ML applications significantly help in mitigating uncertainty and minimizing waste in the food industry. Morariu et al. (2020) proposed a hybrid control solution that integrates ML algorithms with Big Data (BD) techniques for predictive production planning, including operations planning and resource allocation, as well as predictive maintenance. Their study demonstrated that forecasting resource performance metrics such as energy consumption and timeliness and incorporating these insights into production planning improves overall efficiency. Wu et al. (2021) introduced a Supervised Learning-Driven (SLD) heuristic to solve the Capacitated Facility Location and Production Planning (CFLPP) problem. They used a large dataset of small-scale CFLPP instances to train naïve Bayes models. Their findings revealed that the SLD heuristic outperformed the CPLEX solver in terms of solution quality. González Rodríguez et al. (2020) developed a new ML-based methodology for designing an AI-driven decision-making system tailored for production centers within Closed-Loop Supply Chains (CLSC) under uncertainty. This methodology was validated in an industrial hospital laundry case, highlighting its applicability for production centers integrated with CLSC. Gyulai et al. (2014) presented a robust regression-based approach for APP to address capacity analysis and flexible flow assembly lines. They incorporated a multivariate linear function representing capacity requirements into a Mixed-Integer Linear Programming (MILP) model for APP. Using an industry-related dataset, they demonstrated that the regression-based method produced robust production plans capable of managing uncertainties effectively, as validated through discrete event simulation. Mirzapour Al-e-hashem et al. (2013) introduced a stochastic programming model for multi-period, multi-product, multi-site APP in a green supply chain under demand uncertainty. The model incorporates comprehensive cost parameters, quantity discounts, lead time interrelations, emissions, and shortage penalties. By linearizing the nonlinear mixed-integer problem, the model ensures global optimality for its convex structure. A numerical example validates its practical application and highlights its potential for sustainable supply chain management.

Waschneck et al. (2018) applied Reinforcement Learning (RL) using Google DeepMind's Deep Q Network (DQN) agent to production scheduling, demonstrating its potential for Industry 4.0 applications. They implemented the approach in a complex, dynamic production environment and validated it through a small factory simulation of an abstracted front-end-of-line semiconductor production facility, showcasing the system's feasibility. Yu et al. (2018) explored Aggregate Service Planning (ASP) for cloud manufacturing systems, drawing from APP principles. This approach focused on forecasting service demand rather than product demand. Data mining techniques were employed to achieve high-quality predictions. Chen et al. (2020) integrated ML and Model Predictive Control (MPC) to address collaborative production planning challenges. They solved a regression problem to estimate unknown parameters and validated their approach through numerical simulations, confirming its accuracy and effectiveness. Moosavi et al. (2021) conducted a comprehensive review of key Industry 4.0 technologies, including ML, Deep Learning (DL), Internet of Things (IoT), Artificial Intelligence (AI), Cloud Computing, Security, Blockchain, and Big Data (BD), particularly for pandemic management in manufacturing and other sectors. They concluded that these digital transformation technologies serve as accelerators toward Industry 4.0 and can be instrumental in managing future pandemics effectively.

The application of stochastic models and processes for System Dynamics (SD) modeling in production systems remains underexplored, despite its considerable potential. Yin et al. (2003) addressed this gap by modeling capacity processes and random demand in uncertain production systems using two finite-state continuous Markov chains. Their framework aimed to determine the optimal production rate by minimizing expected costs and was applied to the paper industry as a case study. Jamalnia et al. (2017) proposed an integrated approach combining Discrete Event Simulation (DES) with SD modeling, a multi-objective MILP model, and various Multi-Criteria Decision-Making (MCDM) techniques. This methodology was developed to evaluate different APP strategies under uncertainty and was validated using a case study in the soft drink industry. Pérez-Lechuga et al. (2021) employed Markov-chain theory to model production systems, focusing on key variables such as expected production for each machine, total expected production across all facilities, machine idle times, and overall productive efficiency. Islam et al. (2022) developed a two-stage stochastic programming model for APP to handle uncertainties in demand, labor and machine capacity, and power generation. In the first stage, the model addressed these factors, while the second stage examined uncertain hourly electricity loads over a one-year horizon. The final MILP model was tested on a hypothetical energy-intensive production system using CPLEX-AMPL software, demonstrating its effectiveness in managing complex uncertainties. Tirkolaee et al. (2022) tackled a sustainable APP using a hybrid bi-objective MINLP model with a Markov process to manage inventory levels. The model minimized total cost and environmental pollution while capturing system dynamics through a continuous-time Markov chain. Numerical examples demonstrated the method's efficiency, producing optimal solutions within 65 seconds. Sensitivity analyses also highlighted parameter stability, offering practical insights for resource allocation and managerial decision-making under uncertainty. To sum up, a tabular form comparison is provided in Table 2 which clearly demonstrates different directions to study APP in the literature.

Table 2. Table of literature review.

References	Decision-making			Model				Uncertainty				Solution			Extra Features					
	m_1	m_2	m_3	t_1	t_2	t_3	t_4	u_1	u_2	u_3	u_4	s_1	s_2	s_3	e_1	e_2	e_3	e_4	e_5	e_6
Mirzapour Al-e hashem et al. (2013)				•						•		•			•	•	•	•		
Khalili-Damghani and Shahrokh (2014)		•		•				•							•	•		•		
Türkay et al. (2016)				•								•						•	•	
Modarres and Izadpanahi (2016)		•			•				•									•	•	
Gholamian et al. (2016)	•	•		•				•				•						•		
Chaturvedi (2017)					•									•					•	
Entezaminia et al. (2017)				•					•	•		•			•	•	•	•		
Hahn and Brandenburg (2018)					•					•				•				•	•	
Mehdizadeh et al. (2018)						•						•	•		•	•				•
Rasmi et al. (2019)		•		•										•					•	
Goli et al. (2019)		•		•					•			•	•							
Tirkolaee et al. (2019)		•		•				•								•				
Darvishi et al. (2020)							•	•	•			•					•			
Rahmani et al. (2021)		•		•					•			•			•	•		•		
Dohale et al. (2022)	•	•						•				•			•	•		•		
Tirkolaee et al. (2022)		•				•				•		•			•			•	•	
Tirkolaee et al. (2023)	•	•		•				•	•			•			•	•		•	•	
This thesis		•	•			•		•				•			•	•		•		•

Decision Analysis (m_1: MADM, m_2: MODM, m_3: Statistical decision analysis); Model Type (t_1: MILP, t_2: Other Linear, t_3: MINLP, t_4: Other Nonlinear); Uncertainty (u_1: Fuzzy/Possibilistic, u_2: Robust Optimisation, u_3: Probabilistic/Stochastic, u_4: No Uncertainty); Solution Methods (s_1: Exact/Solver, s_2: Metaheuristic, s_3: Heuristic, s_4: No Specific Method); Extra Features (e_1: Multi-product, e_2: Multi-period, e_3: Multi-site, e_4: Case study/Real life example, e_5: Sustainability, e_6: Human factors/Reliability).

2.5. Research Gaps, Motivations and Contributions

As can be comprehended from the above literature review; first, most of the studies on APP have not included environmental and sustainability criteria as part of their modelling approach, but rather focus on economic objectives. On the other hand, human-related issues and factors have been widely neglected, which are the pivot elements of manufacturing systems nowadays as part of sustainable development. Moreover, application of ML and statistical analysis has not been considered by researchers in a way which directly contributes to a more efficient design of APP systems. Finally, reliability has been fully ignored in the literature which, however, is an important part of production engineering, emphasizing the ability of equipment to function without failure.

Against this background and as the main motivation of this research, there must be a decision-making system to deal with the problem complexity considering both qualitative and quantitative factors. To be more specific, a multi-level decision-making system for APP has been built while addressing the reliability, human factors, sustainability and uncertainty aspects that have not been yet addressed at the same time in the literature to the best of my knowledge. In this respect, the proposed decision-making system includes three main stages when it comes to the methodology part. In the first stage, data collection/analysis is done considering a real case study problem of an automotive industry which is mainly focused on different aspects, especially human factors. In the second stage, Multivariate Adaptive Regression Splines (MARS) is utilized as one of the most efficient data science, machine and statistical learning methods to provide the best values for the most important factors.

As a result, a list of fundamental parameters / decision variables is extracted and defined in detail to be incorporated for the mathematical model. In the third stage, a novel bi-objective Mixed-Integer Non-Linear Programming (MINLP) model is developed to formulate a reliable-sustainable APP system under real-life uncertain environment. This model is then treated using Weighted Goal Programming (WGP) and Fuzzy Programming approaches in terms of bi-objectiveness and uncertainty, respectively.

CHAPTER 3
Methodology

3. Methodology

3.1. Research Model

- **MODM Problem Definition**

This study focuses on a multi-period multi-objective sustainable and reliable Aggregate Production Planning (APP) problem. The mathematical model of APP is designed to optimise each scenario individually. It is a multi-objective model operating in a multi-product context. The model aims to reflect real-world conditions and consists of two objective functions. The first objective is to minimise total costs, which include in-house production, outsourcing, workforce, holding, shortage, and employment/unemployment costs. The second objective function is to minimise the absolute relative difference between production and demand; this is equivalent to maximise the "reliability" or "stability" of the APP system (according to *JIT* requirements). Reliability (or stability) depends primarily on the learning and forgetting, maximum efficiency, minimal experience, rest time, fatigue and shift duration.

The model specifically considers the following assumptions:

- A planning horizon is considered.
- Multiple products are taken into account.
- The amount of demand in each period has different values.
- All the costs parameters including production and storage costs and workforce (training) are definitive and certain.
- Inventory shortage is allowed, and this shortage can be satisfied in future periods.
- The possibility of workers overtime and outsourcing is considered.
- Inventory/shortage should be zero at the end of the planning horizon.
- Each type of product is produced in a specific production department.
- The reliability (or stability) of the system is defined as the ability to meet the customers demand based on the *JIT* production policy.

The sets, parameters, and variables of the proposed mathematical model are outlined in Tables 2 and 3.

Table 3. Sets and parameters.

Notations	Descriptions
T	Number of planning times and time horizon (t =1, 2,…, T),
N	Number of products or production departments (n =1, 2,…, N),
J	Number of all work shifts (j=1,2: normal time; j=3: overtime),
K	Number of workforce groups (k = 1, 2,…, K),
LI_{kn}	Minimal experience of a worker from workforce group k to work in department n (in terms of production),
KI_{knj}	Maximum efficiency of a worker from workforce group k to work in department n in work shift j (depending on production rate),

Notations	Descriptions
LE_{kn}	Learning rate of a worker from workforce group *k* in department *n* at the beginning of work,
LF_{kntj}	Forgetting rate of a worker from workforce group *k* in department *n* in period *t* and in work shift *j*,
Bt_j	Rest time given to each worker in work shift j,
λ	Fatigue increase coefficient during the work,
μ_t	Fatigue reduction coefficient during the rest of period *t*,
b_j	Fatigue reduction rate in work shift j,
δ_j	Duration of work shift j,
D_{tn}	Demand of product *n* in period *t*,
C_{tn}^{sc}	Production cost of each unit of product *n* by contractors in period *t*,
C_{tk}^{o}	Cost per man-hour workers of workforce group *k* for working overtime in period *t*,
C_{tn}^{s}	Shortage cost of each unit of product *n* from period *t* to *t* + 1,
C_{tk}^{hire}	Cost per man-hour of employment in period *t* for a worker from workforce group *k*,
C_{tk}^{fire}	Unemployment cost per man-hour in period *t* for a worker from workforce group *k*,
O_t	Overtime capacity (hours) per person in period *t*,
WL_k	Minimum number of workers in workforce group *k*,
m_{njk}	Required man-hour of workforce group *k* to produce each unit of product *n* during normal and overtime working (normal *j*=1,2; overtime *j*=3),
W_{0_k}	Initial number of workers of workforce group *k*,
C_{tn}^{p}	Internal production cost of each unit of product *n* (without workforce) in period *t*,
C_{tk}^{r}	Cost per man-hour of workers of workforce group *k* for working normally in period *t*,
C_{tn}^{h}	Holding cost of each unit of product *n* from the period *t* to *t* + 1,
U_t	Normal capacity (hours) per person in period *t*,
SC_{tn}	Maximum production amount of product *n* allowed to be outsourced in period *t*,
WU_k	Maximum number of workers in workforce group *k*,
$I_{0,n}$	Initial inventory value of product *n*,
TRC_{kn}	Training cost of a worker from workforce group *k* for working in department *n*,
$B_{0,n}$	Initial shortage value in department *n*,
R_{tjkn}	Reliability factor of a worker from workforce group *k* working in department *n* during work shift *j* in period *t*

Table 4. Decision and non-decision variables.

Notations	Description
π_{tn}	Amount of product p delivered to customers in period t time period,
y_{tnj}^{PR}	Amount of production of product *n* in period *t* in work shift *j (*normal work: *j*=1 and *j*=2, and overtime *j*=3),
y_{tn}^{SC}	Amount of production of product *n* outsourced to a contractor in period *t*,
I_{tn}	Remaining inventory of product *n* at the end of period *t*,
B_{tn}	Amount of shortage of product *n* at the end of period *t*,
M_{tnjk}^{R}	Manhours of workers from workforce group *k* working to produce product *n* in period *t* in work shift *j* (normal work: *j*=1 and *j*=2, and overtime *j*=3),
N_{tnjk}^{R}	Number of the workers from workforce group *k* working to produce product *n* in period *t* in work shift *j* (normal work: *j*=1 and *j*=2 and overtime *j*=3),
H_{tk}^{1}	Amount of hired manhours in workforce group *k* at the beginning of period *t*,
H_{tk}^{2}	Number of hired workers in workforce group *k* at the beginning of period *t*,
F_{tk}^{1}	Manhours of unemployed workers from workforce group *k* in period *t*,
F_{tk}^{2}	Number of unemployed workers from workforce group *k* in period *t*,
W_{tn}^{D}	Amount of the delivered products to customers to meet demand of product *n* in period *t*,
W_{tn}^{B}	Amount of the delivered products to customer in period *t* to cover previous shortage of product *n*,
W_{tn}^{p}	Amount of covered demand of product *n* in period *t*, which is supplied by the productions of period *t*,
W_{tn}^{I}	Amount of covered demand of product *n* in period *t*, which is provided by available inventory,
XX_{tjkn}	Binary variables indicating whether at least 1 worker of workforce group *k* is working in period *t* and work shift *j* in department *n*,
QQ_{tjkn}	Amount of production of product *n* in period *t* and work shift *j* by workforce group *k* in department *n* (to produce product *n* in this department),
FG_{jt}	Average amount of fatigue created during work shift *j* before rest time in period *t*,
RG_{jt}	Average amount of fatigue remained after rest time in shift *j* in period *t*.

The objectives of the proposed mathematical APP model is presented as follows:

$$\text{minimise}\quad \mathrm{T}otal\ Cost = \sum_{n\in N}\sum_{t\in T}\sum_{j\in J} c_{tn}^{p}\,(y_{tnj}^{PR}) \tag{1}$$

$$+\sum_{t\in T}\sum_{n\in N} c_{tn}^{sc} y_{tn}^{SC} \tag{2}$$

$$+\sum_{n\in N}\sum_{t\in T}\sum_{j=1}^{2}\sum_{k\in K} c_{tk}^{r} M_{tnjk}^{R} \tag{3}$$

$$+\sum_{n\in N}\sum_{t\in T}\sum_{k\in K} c_{tk}^{o} M_{tn3k} ^{R} \tag{4}$$

$$+\sum_{t\in T}\sum_{n\in N} c_{tn}^{h} I_{tn} \tag{5}$$

$$+\sum_{t\in T}\sum_{n\in N} c_{tn}^{s} B_{tn} \tag{6}$$

$$+\sum_{t\in T}\sum_{k\in K} c_{tk}^{hire} H_{tk}^{1} \tag{7}$$

$$+\sum_{t\in T}\sum_{k\in K} c_{tk}^{fire} F_{tk}^{1} \tag{8}$$

$$+\sum_{n\in N}\sum_{k\in K} TRC_{kn}\, H_{tk}^{2} \tag{9}$$

$$\text{maximise} \quad Reliability \quad \text{(stability)}, \quad \text{i.e. } (<=>)$$

$$\text{minimise} \quad \sum_{t\in T}\sum_{n\in N}\left(\sum_{k\in K}\sum_{j\in J}\left|\frac{(1-FG_{jt})\big(QQ_{tjkn}\big)-D_{tn}}{D_{tn}}\right|\right) \tag{10}$$

subject to

$$I_{t-1,n} + \Big(\sum_{j\in J}\big(y_{tnj}^{PR}\big) + y_{tn}^{SC}\Big) - \pi_{tn} - B_{t-1,n} = I_{tn} - B_{tn} \qquad \forall t \in T, n \in N, \tag{11}$$

$$W_{0_k} + H_{tk}^{2} - F_{tk}^{2} = \sum_{j\in J}\sum_{n\in N} N_{t,njk}^{R} \qquad \forall t \in \{1\}, k \in K, \tag{12}$$

$$\sum_{n\in N}\sum_{j\in J} N_{t-1,njk}^{R} + H_{tk}^{2} - F_{tk}^{2} = \sum_{j\in J}\sum_{n\in N} N_{t,njk}^{R} \qquad \forall t \in T\backslash\{1\}, k \in K,$$

$$(y_{tnj}^{PR}) m_{njk} \le M_{tnjk}^{R} \qquad \forall t \in T, n \in N, j \in J, k \in K, \tag{13}$$

$$\sum_{j\in J}\sum_{n\in N} N_{tnjk}^{R} \le WU_k \qquad \forall t \in T, k \in K, \tag{14}$$

$$\sum_{j\in J}\sum_{n\in N} N_{tnjk}^{R} \ge WL_k \qquad \forall t \in T, k \in K, \tag{15}$$

$$M_{tnjk}^{R} \le U_t N_{tnjk}^{R} \qquad \forall t \in T, n \in N, j \in \{1,2\}, k \in K, \tag{16}$$

$$M_{tnjk}^{R} \le O_t N_{tnjk}^{R} \qquad \forall t \in T, n \in N, j \in \{1,2\}, k \in K, \tag{17}$$

$$y_{tn}^{SC} \le SC_{tn} \qquad \forall t \in T, n \in N, \tag{18}$$

$$H_{tk}^{1} \le H_{tk}^{2}(O_t + U_t) \qquad \forall t \in T, k \in K, \tag{19}$$

$$F_{tk}^{1} \leq F_{tk}^{2}(O_t + U_t) \qquad \forall t \in T, k \in K, \tag{20}$$

$$I_{0,n} - B_{0,n} + \sum_{t=1}^{T} \pi_{tn} = \sum_{t=1}^{T} D_{tn} \qquad \forall n \in N \tag{21}$$

$$\pi_{tn} = W_{tn}^{D} + W_{tn}^{B} \qquad \forall t \in T, n \in N, \tag{22}$$

$$\pi_{tn} = W_{tn}^{p} + W_{tn}^{I} \qquad \forall t \in T, n \in N, \tag{23}$$

$$W_{tn}^{I} \leq I_{t-1,n} \qquad \forall t \in T, n \in N, \tag{24}$$

$$W_{tn}^{B} \leq B_{t-1,n} \qquad \forall t \in T, n \in N, \tag{25}$$

$$W_{tn}^{p} \leq \sum_{j \in J} y_{tnj}^{PR} + y_{tn}^{SC} \qquad \forall t \in T, n \in N, \tag{26}$$

$$I_{T,n} = 0 \qquad \forall n \in N, \tag{27}$$

$$B_{T,n} = 0 \qquad \forall n \in N, \tag{28}$$

$$FG_{1t} = 1 - e^{-\lambda \delta_1} \qquad \forall t \in T, \tag{29}$$

$$FG_{j+1,t} = RG_{jt} + \left(1 - RG_{jt}\right)\left(1 - e^{-\lambda \delta_{j+1}}\right) \qquad \forall j \in \{1, 2\}, \ \ t \in T, \tag{30}$$

$$RG_{jt} = FG_{jt}\, e^{-\mu_t b_j} \qquad \forall j \in J, t \in T, \tag{31}$$

$$QQ_{tjkn} = \ N_{tnjk}^{R} \times \left(\begin{array}{c} LI_{kn} + \\ \left[KI_{knj}\left(1 - \exp\left(\dfrac{-1}{LE_{kn}} XX_{tjkn}\right)\right)\right. \\ \left. \times \exp\left(\dfrac{-1}{LF_{kntj}} XX_{tjkn}\right) \times R_{tjkn}\right] \end{array} \right) \qquad \forall t \in T, j \in J, k \in K, n \in N, \tag{32}$$

$$y_{tnj}^{PR} = \sum_{k \in K} QQ_{tjkn} \qquad \forall t \in T, n \in N, \qquad j \in J, \tag{33}$$

$$N_{tnjk}^{R} \leq M\, XX_{tjkn} \qquad \forall t \in T, j \in J, k \in K, n \in N, \tag{34}$$

$$\begin{gathered} \pi_{tn}, y_{tnj}^{PR}, y_{tn}^{SC}, I_{tn}, B_{tn}, M_{tnj}^{R}, N_{tnj}^{R} \geq 0, \\ H_t^1, H_t^2, F_t^1, F_t^2, W_{tn}^{D}, W_{tn}^{B}, W_{tn}^{p}, W_{tn}^{I} \geq 0, \qquad \forall t \in T, j \in J, k \in K, n \in N. \\ XX_{tjkn} \ \in \{0,1\}, \\ QQ_{tjkn} \geq 0. \end{gathered} \tag{35}$$

Second objective function is also sometimes interpreted as a *"stability"* of the APP system, more precisely as a *"balance"* or an *"equilibrium"* between minimum overproduction and maximum underproduction, together with costs to be minimised, which should also be mathematically stable. In fact, researchers may like to prove this overall "stability" analytically, which however is a hard challenge, or otherwise rather uncover it in terms of *selected* or *partial parametric* dependences through various "sensitivity analyses".

This APP by its second objective function given in Eqn. (10) can be interpreted as a refined problem of constrained *"L_1-regression"* (Aster et al. (2018)). Compared to the more famous *L_2-regression,* also called *least-squares* or *Gaussian estimation*, L_1-regression has the disadvantage of a certain (albeit not excessive) non-differentiability, but also the important advantage of a greater ***robustness*** of the solutions against perturbations in the data or parameters, for example due of outliers.

The first objective function provides the total cost minimisation including first 9 terms.

Constraint (1) shows the production costs within the company.

Constraint (2) states the costs of the outsourcing.

The costs for working normally are represented in Constraint (3).

Constraint (4) corresponds to the overtime costs.

Constraint (5) and (6) indicate the holding and shortage costs, respectively.

Constraint (7) and (8) show the costs of employment and unemployment of the workforce, respectively.

Constraint (9) represents the total costs related to improving the skills of workers at the start of working.

Constraint (10) represents the second objective which minimises the absolute relative difference between production and demand with the goal of maximising the reliability (or stability) of the APP system.

Constraint (11) describes the balance of inventory in each period.

Constraint (12) represents the balance of the workforce in each period.

Constraint (13) specifies the relation between the manhours required in each period using the amount of production.

Constraints (14) and (15) determine the lower and upper limits of the number of workforce to work normally, respectively.

Constraints (16) and (17) limit the number of the required hours for the workforce to work normally and overtime, respectively.

Constraint (18) ensures that the production amount of outsourcing does not exceed the maximum allowed value in each period.

Constraint (19) limits the capacity of hired workforce in man-hours. In other words, the employed workforce along with their number can work in the company to the extent of their maximum capacity.

This is exactly the case which occurs in Constraint (20) for the number of unemployed workforce.

Constraint (21) states that the total amount of products delivered to the customers over the whole periods plus the initial inventory must meet the overall demand for the 4 periods plus the initial amount of shortages.

Constraint (22) calculates the amount of received products from the customers' point of view, which is related to the demand for each period and the amount of shortages occurred in previous periods.

Constraint (23) calculates the amount of delivered products from the company's point of view, which is based on the amount of inventory and production in each period.

Constraint (24) guarantees that the amount of inventory products delivered to the customers should not exceed the available inventory at the beginning of a given period.

Constraint (25) ensures that the amount of products delivered to the customers due to a shortage covering should be less than the amount of that shortage at the beginning of a given period.

Constraint (26) indicates that the amount of delivered products to the customers cannot exceed the total production capacity (for both inside and outside productions).

Constraints (27) and (28) guarantees that the inventory and shortage level should be 0 at the end of the planning.

Constraint (29) calculates the amount of fatigue created during work shift 1 before rest time in each period.

Constraint (30) calculates the amount of fatigue created during work shifts 2 onwards before rest time in each period.

Constraint (31) computes the amount of fatigue remaining after rest time in each work shift and period.

Constraint (32) computes the amount of production done in each period and work shift by each worker in each department.

Constraint (33) shows the relation between the production-amount variables.

Constraint (34) ensures that relationship between the assignment of at least one worker from a given group and the number of workers from that group. (These constraints aim at some non-0 amounts of production in Constraint (32). The "dummy" parameter M is positive and can be suitably tuned by the decision maker. It was not included it in Table 3.)

Constraint (35) specifies the types of the variables.

3.1.1. Fuzzy Programming

Fuzzy logic is a mathematical framework that extends traditional binary logic to process uncertain information. It was first offered by (Goguen, 1973) and ever since been widely applied in various fields such as artificial intelligence and control systems, decision-making and management, engineering and operational research, etc. The main idea behind fuzzy logic is to represent uncertainty through a series of membership functions and linguistic variables (Chen & Pham, 2000). Unlike binary variables, linguistic ones can assume a range of values between 0 and 1, reflecting the degree of membership in some assertion.

Fuzzy logic enables a wide area of applications, ranging from artificial intelligence to operational research and management (OR-MS). Whenever uncertainty is modelled with the help of fuzzy logic, variations in the major parameters of the problem are addressed. Thus, when the parameters assume values in real-life situations, the solutions offered by a fuzzy model are more reliable and robust to parametric variations than the solutions allowed by a deterministic OR-MS model.

In many real-life circumstances, it is hard to make accurate or precise decisions on the basis of the given data. Under such conditions, fuzzy logic can be used to model and process the uncertainty inherent in these situations and make more reliable decisions based on the information available. Among the main applications of fuzzy logic is dealing with incomplete data about the parameter values of decision-making problems in cases of environmental uncertainty, such as at natural disasters (Khalilpourazari et al., 2020a, 2020b). For such emergencies or disasters, there is only a little complete historical data. Hence, it is necessary to obtain expert opinions on probable or possible values of key parameters such as demand and supply in order to make a reliable or stable decision. This is where fuzzy logic can come into play fruitfully.

In this work, fuzzy logic can be employed in production planning (PP) to manage uncertainties related to demand, production capacity, and lead times, and to help managers make production planning and resource allocation decisions.

To study the uncertain nature of demand parameters, a *Fuzzy Linear Programming* (*FLP*) is proposed. To this end, a fuzzy triangular number $\widetilde{D}_{tn} = (D_{tn}^1, D_{tn}^2, D_{tn}^3)$ is defined for the demand of products in period *t*. The membership function is given in Eqn. (36) and Figure 2 as a triangular distribution.

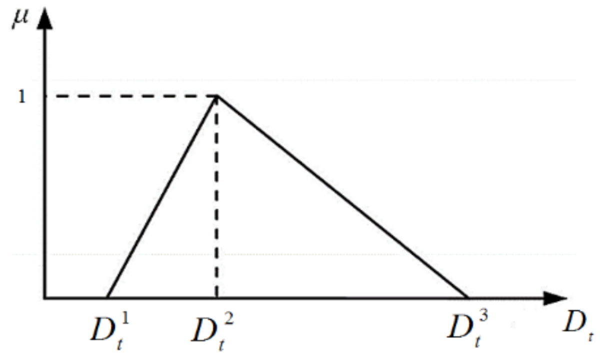


Figure 2. Triangular distribution of fuzzy demand.

$$\mu(D_t) = \begin{cases} 0, & \text{if } D_t \leq D_t^1, \\ (D_t - D_t^1)/(D_t^2 - D_t^1), & \text{if } D_t^1 \leq D_t \leq D_t^2, \\ (D_t^2 - D_t)/(D_t^3 - D_t^2), & \text{if } D_t^2 \leq D_t \leq D_t^3, \\ 1, & \text{otherwise.} \end{cases} \tag{36}$$

Now, based on the ranking approach of Jiménez et al. (2007), all fuzzy parts of the model are converted to their equivalent crisp forms as follows:

$$\begin{aligned} EV_\gamma(\widetilde{RD}) &= \overline{RD} \\ &= (1-\gamma)\left(\left(\frac{\sum_{n\in N}\sum_{t\in T}\left(\sum_{k\in K}\sum_{j\in J}\left|\frac{(1-FG_{jt})(QQ_{tjkn})-D_{tn}^1}{D_{tn}^1}\right|\right)+\sum_{n\in N}\sum_{t\in T}\left(\sum_{k\in K}\sum_{j\in J}\left|\frac{(1-FG_{jt})(QQ_{tjkn})-D_{tn}^2}{D_{tn}^2}\right|\right)}{2}\right)\right) \\ &\quad +(\gamma)\left(\left(\frac{\sum_{n\in N}\sum_{t\in T}\left(\sum_{k\in K}\sum_{j\in J}\left|\frac{(1-FG_{jt})(QQ_{tjkn})-D_{tn}^2}{D_{tn}^2}\right|\right)+\sum_{n\in N}\sum_{t\in T}\left(\sum_{k\in K}\sum_{j\in J}\left|\frac{(1-FG_{jt})(QQ_{tjkn})-D_{tn}^3}{D_{tn}^3}\right|\right)}{2}\right)\right), \end{aligned} \tag{37}$$

$$I_{0,n} - B_{0,n} + \sum_{t=1}^{T} \pi_{tn} = \sum_{t=1}^{n}\left((1-\alpha)\frac{D_{tn}^2 + D_{tn}^3}{2} + \alpha\frac{D_{tn}^1 + D_{tn}^2}{2}\right) \quad \forall n \in N, \tag{38}$$

where Eqns. (37) and (38) are the crisp forms of the second objective (Eqn. (10)) and Eqn. (21), respectively, and $EV_\gamma(\widetilde{RD})$ represents the expected value of $\widetilde{RD}$ based on the parameter $\gamma \in [0, 1]$ indicating the *degree* of *optimism* of a decision maker. The value 0.3 has been assigned for this parameter. Moreover, $\alpha \in [0, 1]$ is the *feasibility degree* of the constraint, which is assigned by the decision maker according to the risk acceptance of the constraint violation (Madadi & Wong, 2014). The considered value for this parameter is 0.8 in this study.

3.1.2. Weighted Goal Programming

One of the most popular and successful approaches for resolving multi-objective decision-making issues in the real world is *Goal Programming* (*GP*). The vast majority of multi-objective decision-making situations are shown to be target centric, requiring the accomplishment of specific objectives. Usually at odds with one another, these ambitions can be conflicting objectives or conflicting values. GP is a strong optimisation method that makes complicated decisions possible while accounting for a range of goals. Its uncomplicated and easy-to-understand methodology makes it an invaluable tool for decision-makers, especially today.

Since there are numerous potential solutions to a multi-objective problem, it is typically challenging to determine which one is optimal. To solve these kinds of problems, when objective functions are in conflict and commensurately related to one another, Charnes et al. (1955) established the concept of GP in order to maximise the degree of achievement of related goals. It was later defined more precisely by Charnes and Cooper (1957).

GP's primary aim was to reduce the gap between performance as expected and as achieved. The classic GP method was presented by Charnes and Cooper (1977) as one of the most applicable multi-objective programming approaches. Due to its ease of use and simplicity, GP has been used in a variety of fields, such as marketing, engineering, locational analysis, economic models, accounting and the financial side of stock management, human resources, academic resources, quality control, production, and operation management (Aouni et al., 2014; Maity and Roy, 2017; Hashemi Doulabi and Khalilpourazari, 2023; Roy et al., 2017).

It was suggested to deal with problems with multiple conflicting objectives. There is a common structure for all GP-based models which tries to minimise unfavourable deviations from the pre-defined values of objective functions. According to the GP philosophy (Yu and Li, 2022; Kuvvetli, 2023), objectives may be fully or partially accomplished, or in certain situations, they may not be met at all. The choice of preferred weights affects the caliber of the outcomes. A new extension of GP known as *Weighted Goal Programming* (*WGP*) was created by combining the idea of weights with conventional GP (Gezen and Karaaslan, 2022). WGP allots *weights* to each objective to distinguish their importance and determine an optimal solution which in a best way satisfies all objectives.

In other words, this approach takes into account ideal (pleasant) levels for the objective functions, which can be defined by decision-maker(s), and seeks to minimise the total deviation from these ideal levels. Three principal elements are required to be regarded in the application of GP:

1. ***System constraints***: represent the resources constraints and the constraints inflicted by the decision conditions.
2. ***Goal constraints***: delineate managerial practices and various quantities of objective functions pursued by decision-makers.
3. ***Objective function***: minimises the deviations from the ideal values defined for each objective with respect to its importance degree (weight).

Furthermore, objective functions are modelled according to two principles:

1. ***Deviation variables***: denotes the variation between the quantities of objective functions to be obtained and their ideal values. In this regard, d_g^+ and d_g^- stand for the positive and negative deviation variables defined for g^{th} objective function, respectively. The aim is to minimise the sum of these positive and negative deviation variables.

2. ***Priority factors***: identify the objective functions to be optimised earlier based on the level of priority.

The general model of *WGP* approach is as follows:

$$\text{minimise} \sum_{g=1}^{G} We_g\left(d_g^+ + d_g^-\right) \tag{39}$$

subject to

$$h_q(X) = (\leq \text{ or } \geq)\ 0 \quad (q = 1,2,\dots,Q),$$
$$f_g - d_g^+ + d_g^- = b_g \quad (g = 1,2,\dots,G),$$
$$d_g^+, d_g^- \geq 0 \quad (g = 1,2,\dots,G).$$

Here, in benefit objectives, negative deviations (d_g^-); and in cost objectives, positive deviations (d_g^+) are to minimised. In Model (39), $h_q(X)$ and b_g denote the q^{th} constraint and the ideal value of g^{th} goal, and f_g stands for the g^{th} goal. The positive and negative deviations are also computed below:

$$d_g^- = \begin{cases} b_g - f_g, & \text{if } f_g < b_g, \\ 0, & \text{otherwise,} \end{cases}$$
$$d_g^+ = \begin{cases} f_g - b_g, & \text{if } f_g > b_g, \\ 0, & \text{otherwise.} \end{cases} \tag{40}$$

Here, We_g shows the positive weight representing the significance of the goals. It must be taken into account that these weights are assigned based on experience and experts' opinions and they should add up to 1 ($\sum_{g=1}^{G} We_g = 1$).

Thus, the following changes are implemented to the model to treat the 2 objective functions at the same time:

$$\text{minimise} \quad WGP = \left(We_1 \frac{d_1^+}{b_1}\right) + \left(We_2 \frac{d_2^+}{b_2}\right). \tag{41}$$

The objective function of WGP model is built up as a weighted sum of positive deviation for the 1st and 2nd objective functions and in order to be of minimisation type, respectively.

Furthermore, the following constraints are included to the set of constraints:

$$TC - d_1^+ + d_1^- = b_1, \tag{42}$$

$$\overline{RC} - d_2^+ + d_2^- = b_2. \tag{43}$$

Here, ideal values ($b_g, g = 1,2$) are specified through optimising the single-objective model with g^{th} objective function.

Finally, the entire model is built based on the Constraint (1)-(9), with Eqn. (37) as a second objective function with constraints (11)–(20), (22)–(35) and (38), and the constraints (42)–(43), such that the merged single-objective function of the model becomes expressed in Eqn. (41).

Therefore, the final model is given as follows:

$$\text{minimise} \quad WGP = \left(We_1 \frac{d_1^+}{b_1}\right) + \left(We_2 \frac{d_2^+}{b_2}\right) \tag{44}$$

subject to

$$TC - d_1^+ + d_1^- = b_1, \tag{45}$$

$$\overline{RD} - d_2^+ + d_2^- = b_2, \tag{46}$$

$$I_{0,n} - B_{0,n} + \sum_{t=1}^{T} \pi_{tn} = \sum_{t=1}^{n} \left((1-\alpha) \frac{D_{tn}^2 + D_{tn}^3}{2} + \alpha \frac{D_{tn}^1 + D_{tn}^2}{2} \right) \qquad \forall n \in N, \tag{47}$$

Constraints (11)-(20), (22)-(35).

3.2. Research Methodology

Selecting an appropriate research methodology is essential since it creates the framework for a reliable and trustworthy study. This study's methodology was well chosen, and it was thoroughly justified. The systematic approach ensures that the many components of the methodology come together to address the research questions and objectives. By aligning different elements of the technique with one another, the research ensures that the data collection, analysis, and interpretation procedures are consistent and coherent. This coherence provides a strong basis for drawing perceptive conclusions and enhances the validity of the study's findings.

3.2.1. Matrix Questionnaires, and Data Gathering and Preparation

In this work, a new academic and managerial approach is introduced and unfolded, illustrated and justified, emanating from its very beginnings in understanding the needs of all sides involved, of their interdependences, of expressing these broadly through a novel "matrix questionnaires", through careful data collection and arrangement, data processing and condensation, data evaluation, interpretation and utilization. A novel "grassroots approach" was developed for implementation. In fact, in the following steps, those preparations are then implemented into the decision-making problem in "Aggregate Production Planning" (APP), which is solved in this research project and is also referred to as *"HF-APP"*.

- **Matrix Questionnaires (MQ1 and MQ2)**

The work for the creation of the questionnaires *MQ1* and *MQ2* is characterized by the best possible fulfillment of the generally recognized "rules". As far as it was even possible, these rules, which were followed, include (cf. Więcek-Janka, 2016):

(a) Rule of gradual move from general to specific questions,
(b) Principles of gradual exhaustion of the topic,
(c) Rules of excitement of interest, and
(d) Helpful questionnaire instructions.

To all this it also belonged:

(i) use of a simple and understandable language,
(ii) formulation of the questions in a way not to imply responses,
(iii) inference from events of a more distant past as much as it is helpful,
(iv) not too much use of relative concepts,
(v) not asking too extensive personal questions,
and in this particular case also,
(vi) clear questions and tasks to the respondents, and
(vii) clear indication of willingness to help the respondents in cases where clarification or support is required.

Of course, such a preparation of the matrix questionnaires *MQ1* and *MQ2* could only happen with the necessary trade-offs within the given overall complexity, and in view of the extremely difficult and intensified conditions of the real-world task. So, all conceivable scientific methods had to be used, along with the human intuition and overall vision.

The matrix questionnaires *MQ1* and *MQ2* based on and reveal the high commitment, including the human-factors-based promise and the vision with regard to the entire task of modeling and solving the APP problem. What should be promising would be an analysis (assessment) of the response behavior of the experts or foremen, of their understanding and inner processing of matrix questionnaires, of the variability of the feelings, attitudes and preferences of the workers or employees, all of this as individually and as comprehensively as possible, and in this context of some essentials about the role of economy, politics and current events in the country of Iran. Then, additional rich variables and data will be obtained, which could allow further conclusions and a meaningful optimal experimental design.

Regarding MQ1: The first questionnaire is deployed in order to collect information regarding *General System*. The questionnaire has 4 criteria and 15 factors together with their sub-factors which are mostly related to the overall situation for the system and workers. For each cell in the questionnaire, one number was asked as the assessment of **an average value for the importance of each factor to each criterion** with respect to employees in the company. The respondents answer on the behalf of workers by considering their situations in the company. As in the case of input variables or factors with their standardized reference interval [0, 10], the respondent for the criteria, namely for matrix cells in the "crosshairs" between sub-factors (or levels) and criteria, was asked to pick values in the interval [0, 10], where 0 means the lowest while 10 represent the highest value. In order to guarantee the respondent an appropriate and sufficient richness when selecting values, it was also allowed the non-integer values of ratios, i.e., rational numbers in general. In *MQ1*, one integer value (5) and two values with one decimal place (0.1 and 9.8) were given as examples for entries in the matrix cells. Despite all these implicitly possible and verbally permitted offers, only integer values were always selected and used. The value 0 was not selected once by the expert or respondent completing *MQ1*.

Regarding MQ2: This questionnaire has been created to observe *Workforce Satisfaction* with respect to the working environment of the workplaces. The questionnaire has 5 criteria and 35 factors together with its sub-levels. For each cell in the questionnaire, one number was asked as the assessment of **how each factor contributes to each criterion (at different levels)** with respect to employees in the company. The respondents answer on the behalf of workers by considering their situations in the company. The selection of the factor scale (2, 4, 6, 8, 10) of levels was made in an equidistant manner in the reference interval [0, 10], where 0 means the lowest while 10 represents the highest value. The levels can also be viewed as representative of the corresponding sub-intervals of [0, 10] of length 2 below such a level, e.g., level 6 for the half-open sub-interval (4, 6]. By choosing the level as the largest value in a sub-interval, the largest value was rewarded, so to speak more positive or optimistic numbers, ultimately with regard to overall human-factor oriented and supported APP approach. What is more, workers, i.e., the humans, have a "voice" and to make it "audible", no matter how quiet might be. So, the "worthless" level 0 was not included here. The equidistance of the levels also facilitates the selection of weighting factors for the levels and the numerical calculations for the "reliability" and "manhour" factors or values. For the 5 criteria in *MQ2,* 4 sub-criteria, R&D, QC, PM and QA, *Research and Development* (*R&D*), *Quality Control* (*QC*), *Production and Manufacturing* (*PM*) and *Quality Assurance* (*QA*)**,** have been retained, which include information about the main fields of work or the 4 basic positions of the surveyed employees. The latter were selected together with the experts or respondents. Here, the autonomy and integrity of the *MQ2* matrix questionnaire itself was always guaranteed. For each of the 5 criteria, gentle weighting factors have been assigned in advance for the orientation and orientation of the respondent. The latter could, but did not have to, resort to this decision-making aid. As can be seen from the insignificant change of value distribution in criteria with low proposed weighting factors, he or she again hardly used this offer. Again, the respondents were asked to select values in the interval [0, 10] for the matrix cells in the "crosshairs" between sub-factors and sub-criteria. The non-integer values of ratios were also allowed, whether rational or irrational. The values 0.1, 5 and 9.8 were given again as examples for entries in the matrix cells. In fact, when filling out the *MQ2* matrix questionnaire, only integer values were selected and the value 0 was left out.

The weight factors are also decided by experts by considering the importance of each criterion for the overall system and the work environment effect on workers. Weight factor values are given for the all criteria in both questionnaires.

3.3. Data Collection and Pre-Analysis

Matrix questionnaires reveal averaged values over all employees in a company or in a division, subdivision, sector or subsector of it. In this regard, the data are "concentrated" already, namely of those employers. Since the project takes place in the automotive industry, it is often used the employers as "worker". It should be kept in mind that this means a common and well-understood simplification.

In the questionnaires, so many criteria with their possibly affecting factors and sub-factors have been used. Criteria are placed in columns while the factors and sub-factors are set in rows. For each of the 9 criteria in total (both from *MQ1* and *MQ2*), the 4 sub-criteria, R&D, QC, PM and QA, *Research and Development* (*R&D*), *Quality Control* (*QC*), *Production and Manufacturing*

(*PM*) and *Quality Assurance* (*QA*), were inserted, which provide information about the main fields of work of 5 *workforce groups* and at the same time about the position of the partner company's employees which were considered and cumulatively assessed in the questionnaires. In fact, these 4 basic positions of the employees and the workforce groups or departments themselves were chosen in early dialogue together with the experts or respondents. Here, the interviewed experts (in total 4 and 1 from each R&D, QC, PM and QA) were provided with all necessary information on understanding the qualitative or verbal grades, levels or expression levels in terms of numerical values, and selected according to their positions and experiences within the company. Contact people in the partner company, i.e., specialists and managers who acted as *respondents* for the matrix questionnaires *MQ1* and *MQ2* were cared for and supervised by the research team. All of these experts are the managers of their departments with at least 10 years of working in the industry. Managers were reached out since they are the decision-makers of their departments and know the details about the departments based on their knowledge.

Moreover, the respondents answer on the behalf of the workers. In total, the number of workers was 64. *5 workforce groups* related to their skill levels and based on their working years (incl. supervisor/production manager, foreman and operators) were defined. There are also *3 work shifts*. All details of the matrix questionnaires were carefully described and plenty of time was given to the interest and attention of the respondent, to questions of understanding and their clear and comprehensive answers.

The data has been collected from Iranian automotive company *Beshel Motors Industrial Company*, ***BMI***, is the largest automotive part manufacturer in Iran (https://www.beshelmotors.com) with 30 years of experience and more than 1000 pieces of daily production. Consequently, the additional data from the questionnaires contains

- the values which experts give on the behalf of workers,
- instead of calibration, expert knowledge (*experimental design*),
- pre-processed information (average values), all numbers are estimated.

On the other hand, there was a remarkably high and diverse numbers of data in the filled-out "matrix questionnaire" just because of the high number of "input variables" or "factors" and the still remarkably high number of "goals" or "criteria". Therefore, for calibrating the HF-APP optimisation problem in a crisp and not too complex way, a "data reduction" and, in fact, "dimension reduction", or "variable reduction" and even "sub-goal reduction" was performed.

In this study, in the obtained data, there was a high number of differences between independent variables and dependent variables. Hence, *MARS* could not obtain basis functions via obtained data set, consequently could not receive any model by using just real-world data sets. Whenever on the one hand the dataset could have become too small - for instance, since any "employee data" so to speak disappeared in the "fold" of an average number, i.e., they were statistically "masked" – ***simulation*** will be used as a method of data acquisition (Jäckel, 2002).

Finally, it is worth emphasizing once again that the matrix questionnaires *MQ1* and *MQ2* are efficiently used for data collection, and it will systematically examine and modelled the relationships between the factors or input variables and the criteria or output variables using the method of choice, *MARS* (*Multivariate Adaptive Regression Splines*), including a respective determination and identification of the most important factors. In a large number of scientific studies under this broader research scope, MARS method has proven to be refineable with new variants, flexible scientifically and in application, and over all highly competitive compared to other methods of statistics, statistical learning, machine learning or artificial intelligence. MARS-based methods have the inherent ability to *capture complex, nonlinear relationships* between variables, overcoming the limitations of traditional linear regression models. This allows to uncover complicated patterns in the data that would otherwise be elusive. As *J. Friedman* and his colleagues have shown (Hastie et al. 2009), MARS consistently outperforms traditional linear models in terms of prediction accuracy. The use of base functions and splines ensures that MARS models adapt to the nuances of the data, resulting in superior statistical performance. Detailed information about MARS, its conic and robust versions and their applications can be found in Taylan et al. (2010), Weber et al. (2012), Özmen et al. (2012, 2013, 2014a, 2014b, 2014c, 2023) and Graczyk-Kucharska et al. (2020a, 2020b, 2022, 2023) and Szafrański et al. (2022).

In the next section of this PhD. thesis, MARS methodology will be discussed in more detail.

3.3.1. MARS

Multivariate Adaptive Regression Spline (*MARS*) is an advanced and appreciated and increasingly used technology from statistics, statistical learning and machine learning that is important in regression and classification (Friedman, 1991; James et al., 2023). It has allowed for more and more applications in numerous fields of science and management, economy and environmental sciences, engineering and medicine, social sciences and the humanities. MARS is very helpful for challenges of modeling and analysis in highly dimensional spaces and it is becoming a premium or first-choice tool for data-fitting with nonlinear und multivariate model functions. A particular advantage of MARS consists in its potential to assess the contributions of a model's basis functions which permits that the additive and the interactive effects of the predictors can serve in the represention of the response variable. The algorithm, code or procedure of MARS, for finding the optimal model function consists of 2 sub-algorithms, referred to as the *forward step* and the *backward step*.

For the sake of a more elegant exposition, a modeling problem will mostly be formulated as a problem from regression. However, classification problems can also be treated as regression problems, even in a unified modeling framework characterized by the basis functions, be it in regression or in classification. A finite dataset typically originates from different kinds of experiments or observations, records or questionnaires - as in this thesis - or a preprocessing of information obtained, e.g., by clustering methods. Various kinds of technologies, be they instruments, based from arithmetics or mental processing through experts - as in this thesis - could help to gain or raise, extract or collect the data.

For this concept of the MARS model, it was was by *Friedman* (1991) who proposed a procedure (Graczyk-Kucharska et al., 2020) as a flexible strategy for high-dimensional probems of nonparametric regression, established on a modified algorithm of recursive partitioning. MARS is established on expansions in pairs of piecewise linear basis functions of the following form:

$$c^{+}(x,\tau)=[+(x-\tau)]_{+}, \qquad c^{-}(x,\tau)=[-(x-\tau)]_{+}. \tag{48}$$

Here, $[q]_{+} := \max\{0,q\}$, whereas τ is a univariate knot. Each such a *truncated* function is piecewise linear, with a knot at the value τ , and both together designate a *reflected pair*. For an illustratioon, Figure 3 is demonstrated.

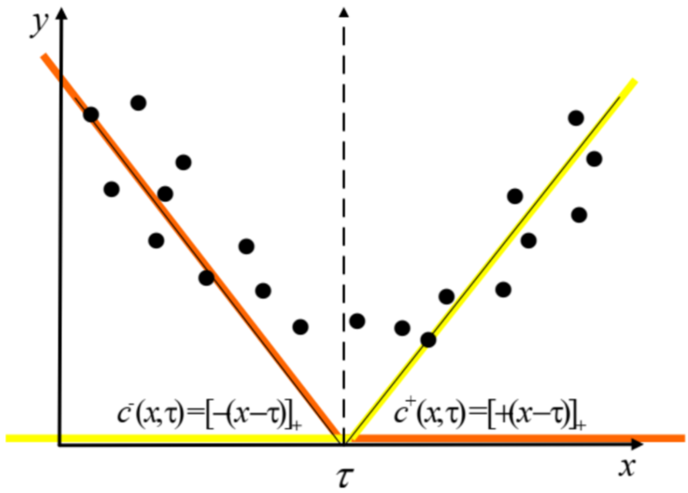


Figure 3. A reflected pair of 1-dimensional basis functions in MARS regression.

The dots in Figure 3 stand for the data $(\bar{\boldsymbol{x}}_i, \bar{y}_i)$ $(i=1,2,...,N)$, reflecting a p-dimensional inputs values of the variable $\boldsymbol{x}$ and the corresponding 1-dimensional responses value of the variable y.

The following general model on the the relation between input and response may be addressed:

$$Y = f(\boldsymbol{X}) + \varepsilon, \tag{49}$$

where Y is a response (random) variable, $\boldsymbol{X} = (X_1, X_2, \ldots, X_p)^T$ is a vector of input (random) variables, and ε is an additive (random) "noise" component, assumed to have 0 mean and finite variance.

First of all, the modeling purpose consists in the construction of reflected pairs for any input X_j $(j=1,2,...,p)$ with p-dimensional knots $\boldsymbol{\tau}_i = (\tau_{i,1}, \tau_{i,2}, ..., \tau_{i,p})^T$ at - or close to - each the input data vectors $\bar{\boldsymbol{x}}_i = (\bar{x}_{i,1}, \bar{x}_{i,2}, ..., \bar{x}_{i,p})^T$ $(i=1,2,...,N)$ of the said input. That close localization is nothing else than a slight technical modification: it can be assumed that $\tau_{i,j} \neq \bar{x}_{i,j}$ for any i and j without loss of generality and to prevent from nondifferentiability in subsequent analyses and refinements (such as analytic sensitivity analyses). The knot values $\tau_{i,j}$ could even be selected farther away from the input values $\bar{x}_{i,j}$, whenever any such a localization might be a chance for a better data fitting.

Based on these 1-dimensional preparations, the set of basis functions reads as follows:

$$\wp := \left\{(X_j - \tau)_+, -(-X_j \ \ -)_+ | \tau \in \left\{x_{1,j}, x_{2,j}, \ldots, x_{N,j}\right\},\ j \in \{1,2,\ldots,p\}\right\}. \tag{50}$$

Whenever all input values deviate from each other, overall there are $2Np$ basis functions. Thus, the function $f(\boldsymbol{X})$ can be represented by a linear combination which is successively built up by the set $\wp$ and with the intercept θ_0, such that Eqn. (49) takes the form

$$Y = \theta_0 + \sum_{m=1}^{M} \theta_m \, \psi_m(\boldsymbol{X}) + \varepsilon. \tag{51}$$

Here, ψ_m $(m = 1,2,\ldots,M)$ are basis functions from $\wp$ or products of 2 or more such functions, ψ_m is taken from a set of M linearly independent basis elements, and θ_m are the unknown coefficients for the mth basis function ($m = 1,2,\ldots,M$) or for the constant 1 itself ($m = 0$). For any of the dimensions corresponding to an input variable, a set of qualified knots $\tau_{i,j}$ is separately assigned and selected to approximate the input levels represented in the data. By multiplying some already existing basis function with a truncated linear function that involves a new variable, *interaction basis functions* are created. Then, the existing basis function as well as the newly introduced interaction basis function are employed in the MARS approximation. Given the data $(\bar{\boldsymbol{x}}_i, \bar{y}_i)$ $(i = 1,2,\ldots,N)$, the mth basis function has the following form:

$$\psi_m(\boldsymbol{x}) := \prod_{j=1}^{K_m} [s_{\kappa_j^m} \cdot (x_{\kappa_j^m} - \tau_{\kappa_j^m})]_+, \tag{52}$$

where K_m is the number of truncated 1-dimensional basis functions which are multiplied for the mth basis function, $x_{\kappa_j^m}$ is the input variable corresponding to the jth truncated linear function within the mth basis function, $\tau_{\kappa_j^m}$ is the knot value corresponding to the variable $x_{\kappa_j^m}$, and $s_{\kappa_j^m}$ designates +1 or −1, the the sign generated.

A *lack-of-fit* criterion reflects the trade-off between (desirable) accuracy and (undesirable) complexity or (wanted) stability. It is applied to assess and compare the possible basis functions. The search for new basis functions can be bounded from above to interactions of a maximum order (or degree). For example, if not more than 2 factors are permitted in interaction, then the bound of $K_m \leq 2$ could be imposed in Eqn. (52).

MARS procedure for determing the model function $f(\boldsymbol{x})$ consists of 2 sub-algorithms or "steps" (Taylan and Weber, 2019):

(i) *Forward step*: Here, forward stepwise search for the basis function takes place with the constant basis function, the just a single one present initially. At each step, the split that minimized some *"lack of fit" criterion* from all the possible splits on each basis function is chosen. The process stops when a user-specified value, $M_{\max}$ is reached. At the end of this process, there will be a big expression in Eqn. (51). Such a model typically overfits the data and so a *backward* deletion procedure is applied.

(ii) *Backward step*: The purpose of this algorithm is to prevent overfitting by reducing the complexity of the model without degrading the fit to the data. Hence, the backward step includes a removal of basis functions from the model which contribute to the smallest slope of the residual squared error at any stage, producing an optimally estimated model $\hat{f}_\alpha$ with respect to any number α of terms. Here α represents some *complexity* of the estimation. To assess the optimal value of α, *generalized cross-validation* (*GCV*) can be employed reflecting the balanced, weighed or relative *"lack of fit"* when using MARS. The criterion GCV is defined by

$$GCV := \frac{\sum_{i=1}^{N}(y_i - \hat{f}_\alpha(\boldsymbol{x}_i))^2}{(1-\frac{\boldsymbol{M}(\alpha)}{N})^2}. \tag{53}$$

Here, $\boldsymbol{M}(\alpha) := u + dM$. Here, N stands for the number of data, M represents the number of linearly independent basis functions, $\boldsymbol{M}(\alpha)$ stands for the number of knots chosen in the forward step, and the parameter d represents some "cost" for basis-function optimization and a smoothing factor or multiplier of the procedure.

This study benefits from the program package of MARS called *Salford MARS* (cf. *Data Mining, Machine Learning & Predictive Analytics Software | Minitab*, n.d.).

3.4. Research Results

3.4.1. Data Exploration

Data on the impact of the general work environment and human factors were gathered using two matrix questionnaires with rating-scale responses. From 2 questionnaires (*MQ1* and *MQ2*), there are 50 factors and 9 criteria as listed in Tables 5-8.

Table 5. Factors from *MQ1*.

Code	Description
$x1_1$	Importance of Age
$x2_1$	Importance of Gender
$x3_1$	Importance of Marital Status
$x4_1$	Importance of Educational Level
$x5_1$	Needed Technical or Professional Skills
$x6_1$	Needed General Skills
$x7_1$	Needed Common Skills
$x8_1$	Needed Qualifications
$x9_1$	Effect of Workplace Distance to Home
$x10_1$	Effects of Employment Type
$x11_1$	Effects of Place of Work

Code	Description
$x12_1$	Effects of Shift Work
$x13_1$	Importance of Training and Education
$x14_1$	Effect of Employees' Personalities
$x15_1$	Importance of Level of Employees' Salary

Table 6. Factors from *MQ2*.

Code	Description
$x1_2$	Satisfaction with sense of achievements
$x2_2$	Satisfaction by being well-matched with working position
$x3_2$	Sense of integration /belonging into the working process / community
$x4_2$	Satisfaction with the trust felt by the managers in general
$x5_2$	Satisfaction with respect for esteem needs by managers
$x6_2$	Satisfaction with current maintenance of the workplace
$x7_2$	Satisfaction with access to materials and equipment
$x8_2$	Occupational health and safety
$x9_2$	Space, lighting and ventilation
$x10_2$	Ergonomic / physical arrangement of work area
$x11_2$	Hygiene and sanitary at the workplace
$x12_2$	Physical health and mental health first aid
$x13_2$	Private health and accident insurance benefits
$x14_2$	Social security benefits
$x15_2$	Professional atmosphere / availability to share experience
$x16$	Communication channel opportunities in the company
$x17$	International relationships between employees
$x18$	Communication style of managers with employees
$x19$	Reasonable expectations of manager
$x20$	Respect / consideration for expectations of workers by managers
$x21$	Trust in director and company generally
$x22$	Satisfaction with working time
$x23$	Courtesy for private issues by manager
$x24$	Company attitudes during crisis time
$x25$	Satisfaction with amount of payment
$x26$	Flexible and home working
$x27$	Forms of work organization
$x28$	Punishment and reward system
$x29$	Strategies to resolve conflicts among employees
$x30$	Treating employees fairly and equally
$x31$	Selection of the right persons for the right positions
$x32$	Proper personality analysis of workers in general
$x33$	Recreational facilities

Code	Description
$x34$	Training programs to enhance work performance
$x35$	Career opportunities

Table 7. Criteria for *MQ1*.

Criteria for ***MQ1***	
Y_1	Overall Fulfilment of Duties
Y_2	Quality of Contribution
Y_3	Production Level
Y_4	Flexibility at Work

Table 8. Criteria for *MQ2*.

Criteria for ***MQ2***	
Y_1	Job Satisfaction
Y_2	Health at Risk
Y_3	Relationship with co-workers
Y_4	System at Work
Y_5	Human Resource Management Operations

As mentioned before, these factors have been investigated through MARS analysis in order to see the important ones for the related criterion. In the next part, the results from MARS analysis will be represented.

3.4.2. MARS Results

In this sub-section, MARS results for *MQ1* and *MQ2* will be represented separately. Highest importance of input variables will be shown for each related criterion. Of course, these variables need not be significant or highly significant for all of those sub-criteria, but for one or several ones.

The good news from *MQ1* is that the results confirm and "reconstruct" the high and supreme importance that have been already placed on education, learning and skills, e.g., in the APP optimization problem and model with its reference to learning and fatigue in the reliability concept. In Tables 9-12, the results for 15 factors and 4 related criteria from *MQ1* are demonstrated.

Table 9. MARS results: Overall fulfillment.

Code	Value
$x4_1$	100
$x3_1$	87.0749
$x10_1$	81.9233
$x13_1$	58.9758

Code	Value
$x11_1$	47.7607
$x2_1$	39.5186
$x6_1$	27.0964

Table 10. MARS results: Quality of contribution.

Code	Value
$x6_1$	100.0000
$x13_1$	91.2921
$x11_1$	82.5667
$x9_1$	36.5221
$x5_1$	25.9007

Table 11. MARS results: Production level.

Code	Value
$x1_1$	100.0000
$x5_1$	84.3177
$x15_1$	66.9343
$x11_1$	59.6161
$x8_1$	32.6083

Table 12. MARS results: Flexibility at work.

Code	Value
$x2_1$	100.0000
$x6_1$	90.2042
$x9_1$	62.5093
$x14_1$	48.2561
$x12_1$	36.8639

Results from *MQ2*, including 35 factors and 5 criteria, are also outlined in Tables 13-17.

Table 13. MARS results: Job satisfaction.

Code	Value
$x16$	100.0000
$x9_2$	100.0000
$x6_2$	70.7857

Table 14. MARS results: Health at risk.

Code	Value
$x35$	100.0000

$x3_2$	69.0903
$x7_2$	46.7069

Table 15. MARS results: Relationship with co-workers.

Code	Value
$x15_2$	100.0000
$x5_2$	100.0000
$x27$	100.0000
$x11_2$	25.9215

Table 16. MARS results: System at work.

Code	Value
$x30$	100.0000
$x24$	100.0000
$x5_2$	53.8356
$x8_2$	53.8356

Table 17. MARS results: Human resource management operations.

Code	Value
$x21$	100.0000
$x9_2$	43.6766

Now, it is possible to create the list of important factors for the *MQ1* part as represented in Table 18.

Table 18. Final important factors out of *MQ1*.

Code	Description
$x1$	Importance of age
$x2$	Importance of gender
$x3$	Importance of marital status
$x4$	Importance of educational level
$x5$	Needed technical or professional skills
$x6$	Needed general skills
$x10$	Effects of employment type
$x11$	Effects of place of work
$x13$	Importance of training and education

For the *MQ2* part, the results support this APP model with highest importance value to the variables related to not only from tangible aspects, such as working conditions or conditions of workers, but also from intangible aspects which might affect workers mentally. The results obtained from 35 factors and 5 related criteria are displayed in Table 19.

Table 19. Final important factors out of *MQ2*.

Code	Description
$x5_2$	Satisfaction with respect for esteem needs by managers
$x6_2$	Satisfaction with current maintenance of the workplace
$x9_2$	Space, lighting and ventilation
$x15_2$	Professional atmosphere / availability to share experience
$x16$	Communication channel opportunities in the company
$x21$	Trust in director and company generally
$x24$	Company attitudes during crisis time
$x27$	Forms of work organization
$x30$	Treating employees fairly and equally
$x35$	Career opportunities

From these results, another major advantage and novelty of the statistical analysis of this work, as condensed by its two concluding *"Score Tables"*, is that the percentage values within the 2 tables can be viewed as *"generalized correlation coefficients"*. By this way, it can be seen the effect of each factor holistically in the system. It has been involved the importance level of each factor for each criterion (if there is any) through the weight factor effect.

Furthermore, these coefficients in the scoring matrix permit to "switch" between rows and columns, so to speak, from output variables to input variables (and vice versa). This allows instead of including all output variables as sub-criteria in the APP's objective function (high complexity and instability), to account for them by including some main corresponding input variables anywhere in the APP (objective function and constraints), thus acknowledging their importance. Tables 20 and Table 21 show the Score Matrices for *MQ1* and *MQ2*, respectively.

Table 20. Detailed results obtained from *MQ1*: *Score Matrix 1*.

Results from MQ1					Scores	Hits (Number of Successes)
Code	**Overall Fulfilment of Duties**	**Quality of Contribution**	**Production Level**	**Flexibility at Work**		
	WF_1: 0.298	WF_2: 0.226	WF_3: 0.231	WF_4: 0.245		
$x1_1$			100%		**23.1**	**1**
$x2_1$	40%			100%	**36.42**	**2**
$x3_1$	87%				**25.926**	**1**
$x4_1$	100%				**29.8**	**1**
$x5_1$		26%	84%	27%	**31.895**	**3**
$x6_1$	27%	100%		90%	**52.696**	**3**
$x8_1$			33%		**7.623**	**1**

Code	Results from MQ1				Scores	Hits (Number of Successes)
	Overall Fulfilment of Duties	Quality of Contribution	Production Level	Flexibility at Work		
	WF_1: 0.298	WF_2: 0.226	WF_3: 0.231	WF_4: 0.245		
$x9_1$		36%		63%	**23.571**	**2**
$x10_1$	82%				**24.436**	**1**
$x11_1$	48%	83%	60%		**46.922**	**3**
$x12_1$				37%	**9.065**	**1**
$x13_1$	59%	91%			**38.148**	**2**
$x14_1$				48%	**11.76**	**1**
$x15_1$			67%		**15.477**	**1**
Sums	443	336	344	365	1488	

Below example calculations show how the weight factors and scores are assigned.

Weight Factors: $WF_1 = \frac{443}{1488} = 0.298$, $WF_2 = \frac{336}{1488} = 0.226$.

Scores: 36.42 = (0.298*40) + (0.245*100) = 36.42 & **31.895** = (0.226*26) + (0.231*84) + (0.245*27) = 31.895.

Table 21. Detailed results obtained from *MQ2*: *Score Matrix 2*.

Code	Results from MQ2					Scores	Hits (Number of Successes)
	Job Satisfaction	Health at Risk	Relationship with Co-workers	System at Work	HRM Operations		
	WF_1: 0.214	WF_2: 0.170	WF_3: 0.257	WF_4: 0.243	WF_5: 0.113		
$x3_2$		69%				**11.73**	**1**
$x5_2$			100%	54%		**38.82**	**2**
$x6_2$	71%					**15.194**	**1**
$x7_2$		47%				**7.99**	**1**
$x8_2$				54%		**13.122**	**1**
$x9_2$	100%				44%	**26.372**	**2**
$x11$			26%			**6.682**	**1**
$x15$			100%			**25.7**	**1**
$x16$	100%					**21.4**	**1**
$x21$					100%	**11.3**	**1**
$x24$				100%		**24.3**	**1**
$x27$			100%			**25.7**	**1**

Code	Results from MQ2					Scores	Hits (Number of Successes)
	Job Satisfaction	Health at Risk	Relationship with Co-workers	System at Work	HRM Operations		
	WF_1: 0.214	WF_2: 0.170	WF_3: 0.257	WF_4: 0.243	WF_5: 0.113		
*x*30				100%		**24.3**	**1**
*x*35		100%				**17**	**1**
Sums	271	216	326	308	144	1265	

The same methods have been used for the calculations of weight factors and scores in *MQ2* as well.

Weight Factors: $WF_1 = \frac{271}{1265} = 0.214$, $WF_2 = \frac{216}{1265} = 0.170$.

Scores: 38.82 = (0.257*100) + (0.243*54) = 38.82 & **26.372** = (0.214*100) + (0.113*44) = 26.372.

As the result of *Scoring Matrix* application, it was possible to select the most important variables with the impact which is related whole system. The percentage values used in the scoring matrix are the values which were obtained from the MARS analysis. These values have been already represented in the Tables 9-17 in a more detailed way. The values in these tables represent the importance of the factors for a single related criterion. That is why, scoring matrix application was essential to create in order to see the importance of each impactful factor with its effect on the whole system. This helps to evaluate the factors more in general and make the results fairer for each factor. Table 22 shows list of the most impactful factors after *MARS* and *Scoring Matrix* applications.

Table 22. Results of *Scoring Matrix*.

Code	Description
$x5_1$	Needed technical or professional skills
$x6_1$	Needed general skills
$x11_1$	Effects of place of work
$x13_1$	Importance of training and education
$x5_2$	Satisfaction with respect for esteem needs by managers
$x9_2$	Space, lighting and ventilation
$x15_2$	Professional atmosphere / availability to share experience
*x*21	Trust in director and company generally
*x*27	Forms of work organization
*x*30	Treating employees fairly and equally
*x*35	Career opportunities

From the results of *MQ2* in the *Score Matrix 2*, the factors *x*24: *Company attitudes during crisis time*, and *x*16: *Communication channel opportunities in the company*, were not included for the APP implementation part even though they also have higher impact value than some other selected factors, because factor $x16$ is already very similar to factor $x15_2$, and $x24$ seems to have a little different concept than APP-HF problem according to the given definition in Chapter 1. So, it was decided to exclude these parameters for this time. Likewise, for the results of *MQ1* in the *Score Matrix 1*, the similar ones to other selected factors were eliminated and the factors with the importance level over 30% were selected in general for the parameter inclusion part. For all the factor selection steps, the main reference was the MARS results. None of the factors were included unless MARS results did not demonstrate them as impactful.

In the next steps, new parameters will be created based on the final most important factors and then included into APP in order to achieve *APP-HF* concept.

In this concept, 3 **key** parameters, *manhour related* - $\boldsymbol{m_{nj}}$, *training cost related* - $\boldsymbol{TRC_{kn}}$, and *reliability factor* - $\boldsymbol{R_{tjkn}}$, were created and implemented into the APP model through goal functions and constraints. Definitions for the parameters will be given in a detailed way in further parts.

- ***m* and *TRC* Parameters**

For this purpose, using the variables $x9_2$ (*space, lighting and ventilation*), $x15_2$ (*professional atmosphere / availability to share experience*) and $x27_2$ (*forms of work organization*), the rate of *manhour-*m_{nj} parameter of each product is calculated via the formula below:

$$m_{nj} = m0 + \left(xx9_2\, m1_{nj} + xx15_2\, m2_{nj} + xx27_2\, m3_{nj}\right), \tag{54}$$

where $xx9_2, xx15_2$ and $xx27_2$ are the *normalized* values of 3 of the most effective factors:

I. $xx9_2 = x9_2 / (x9_2 + x15_2 + x27_2)$,
II. $xx15_2 = x15_2 / (x9_2 + x15_2 + x27_2)$,
III. $xx27_2 = x27_2 / (x9_2 + x15_2 + x27_2)$.

Here, $xx9_2, xx15_2$ and $xx27_2$ are the *normalized* values of 3 of the most effective factors $x9_2, x15_2$ and $x27_2$. The values for $x9_2, x15_2$ and $x27_2$ were calculated according to the values given by the respondents in the MQ2 while $m0$ were obtained from the company in order to determine the value for m_{nj} ($m1, m2$ and $m3$ were generated based on $m0$ and as a result of multiplication of their given weight factors with $m0$). Moreover, the calculations of $x9_2, x15_2$ and $x27_2$ have been performed based on linear equations consisting of the summation of the values from the questionnaire multiplied by the related weight factors. More details of the calculations can be found as an excel sheet form in *Appendix E*.

Now Table 23 presents the corresponding modified manhour-related parameters with their explicit definitions.

Table 23. Modified manhour-related parameters.

Parameter	Description
$m0_{nj}$	The most desirable or "ideal" required manhours of workforce group *k* to produce each unit of product *n* during normal and overtime working which is not affected by human factors; in other words: if a robot is considered towards a worker, or when, in terms of manhours, the relevant employees are working under most desirable or "ideal" conditions, this parameter will make sense.
$m1_{nj}$	A percentage of $m0_{nj}$ to produce each unit of product n by workforce group k during normal and overtime working considering most desirable or "ideal" space, lighting and ventilation. It is defined as $(1-SLV)\, m0_{nj}$, where $SLV \in [0, 100\ [\%]]$ represents the current status of space, lighting and ventilation at the company.
$m2_{nj}$	A percentage of $m0_{njk}$ to produce each unit of product n by workforce group k during normal and overtime working considering most desirable or "ideal" professional atmosphere / availability to share experience. It is defined as $(1-PAA)\, m0_{nj}$, where PAA$\in [0, 100\ [\%]]$ represents the current status of professional atmosphere / availability to share experience at the company.
$m3_{nj}$	A percentage of $m0_{njk}$ to produce each unit of product n by workforce group k during normal and overtime working considering most desirable or "ideal" forms of work organization. It is defined as $(1-FWO)\ m0_{nj}$, where $FWO \in [0, 100\ [\%]]$ represents the current status of forms of work organization at the company.

In the case of the training costs, marked with TRC_{kn}, it is formally similar to the case of m_{nj}-manhours although the interpretations of the content will be somewhat different. In particular, however, the parameters of the *TRC*-factor or -value come from *MQ1*, especially from education and skills aspects, whereas those of the *m*-factor or -value come from *MQ2*.

TRC_{kn} - the training cost of each workforce has been calculated by using the variables $x5_1$ (*needed technical or professional skills*), $x6_1$ (*needed general skills*), $x11_1$ (*effects of place of work*) and $x13_1$ (*importance of training and education*). In fact, for the *TRC*-factor or -value, the equation is as follows:

$$TRC_{kn} = xx6_1\ TRC0_{kn} + (xx13_1 + xx5_1)\, TRC1_{kn} + xx11_1\, TRC_{2kn}, \tag{55}$$

where the *normalized* values of 4 of the most effective factors are given below:

I. $xx6_1 = x6_1 \,/\, (x6_1 + x5_1 + x13_1 + x11_1)$,
II. $xx5_1 = x5_1 \,/\, (x6_1 + x5_1 + x13_1 + x11_1)$,
III. $xx13_1 = x13_1 \,/\, (x6_1 + x5_1 + x13_1 + x11_1)$,
IV. $xx11_1 = x11_1 \,/\, (x6_1 + x5_1 + x13_1 + x11_1)$,

such that $xx5_1,\ xx6_1,\ xx11_1$ and $xx13_1$ are the *normalized* values of 4 of the most effective factors: $x5_1,\ x6_1,\ x11_1$ and $x13_1$. The values for $x5_1,\ x6_1,\ x11_1$ and $x13_1$ were calculated according to the values given by the respondents in the MQ1, while $TRC0_{kn}$, $TRC1_{kn}$ and $TRC2_{kn}$ values have been obtained from the company in order to determine the value for TRC_{kn}. The calculations of $x5_1$, $x6_1$, $x11_1$ and $x13_1$ have been made based on the linear equations which include the multiplication of the values from the questionnaire with the related weight factors, followed by summation. More details of the calculations are provided as an excel sheet form in *Appendix E*.

Table 24 presents the corresponding modified training-cost related parameters.

Table 24. Modified training-cost related parameters.

Parameter	Description
$TRC0_{kn}$	General training cost for workforce group *k* for working in department *n*
$TRC1_{kn}$	Professional training cost for workforce group *k* for working in department *n*
$TRC2_{kn}$	Training cost for workforce group *k* based on the place of works for working in department *n*

- ***R*** **- Reliability Factor and its Reference Value**

R_{tjkn} was first modelled and calculated by using the variables $x5_2$ (*satisfaction with respect for esteem needs by managers*), $x21$ (*trust in director and company generally*), $x30$ (*treating employees fairly and equally*) and $x35$ (*career opportunities*). More details of the calculations can be provided as an excel sheet form in *Appendix E*.

In fact, for the R_{tjkn}-factor or -value, the basic equation is as follows:

$$R_{tjkn} = x5_2 + x21 + x30 + x35. \tag{56}$$

This was done according to any time *t*, work shift *j*, product *n* and especially any workforce group *k*, consisting of employees with the same number of years of experiences and, if possible, any further homogeneity inside of each group or any other similarities between the members of the group.

R will be called as its numerical realizations or samples, a "*specific reliability value*" or *"specific reliability factor"*, often dropping the word *"special"* (in short: *SRF* or *R*-factor, and *SRV* or *R*-value). The Iranian partner from the company were later asked for their *"individualized"* values of *R*, i.e., R_{tjkn}, within a certain scale (in the interval [0, 3]) according to the parameters in the definition of the *R*-value, with help for understanding and possible guidance provided to them in the form of a *reference* (*R*-) *value*.

There are 3 basic scientific approaches for the *"reference value"* of *R*-value, 2 of which are shown here for better understanding

Reference value

***R-value 1*:** Here, 4 sums are built, according to the 4 variables which define the *R-value*. Each of these 4 sums comes from the summation of all its values (cf. *MQ2* file, for example, the sum of the values for $x5_2$ is equal to 388), then to add up these 4 numbers ($x5_2, x21_2, x30_2, x35_2$), and then to translate this sum to a percentage out of 4000 (= $5 * 4 * 5 * 4 * 10$) where 5 is number of criteria, 4 is number of respondents, 5 is number of weight factors, 4 is number of factors, 10 is the highest value, and to compare this percentage with 33.333...% ("= 1/3"), i.e., with the value 1 in the interval [0, 3].

In this case, $\frac{388+416+440+440}{4000} = 0.421$.

This percentage equals to $1{,}264$ ($\in [0, 3]$), when 33,3…% is considered as 1.

For the practical purposes and the exchange with the expert in the company in Iran, the value 1.264 is rounded and taken as ***R-value 1*** = **1.3**.

This value is bigger - showing a higher response towards 4 input variables of workforce satisfaction - than the neutral unit of multiplication, 1, i.e., 33.3…%.

An *advantage* of this *R-value 1* itself is that it creates a "space for improvement" in the sense that the expert approached can (psychologically) sense a freedom to use this "space" (i.e., subinterval of the interval [0, 3]), while it can (but need not) support a distribution of his/her assessments (values in [0, 3]) with an accumulation around a value higher than the *R*-value 1, thus, with a peak rather than a kind of a uniform distribution.

***R-value 2*:** This version comes from *R-value 1* when the *weight factors* (their sum can but need not be 1 here) were used, which naturally chosen as 0.2, 0.4, 0.6, 0.8, 1, for the 5 levels of each of the 4 variables, i.e., 2, 4, 6, 8, 10. Then, one really gets *a "Specific Reliability Factor"*, which is a main advantage. The reference number of 4000 from *R-value 2* has to be replaced by 2400 (4000 x 0,6, where 0,6 = 1/5 (0.2 + 0.4 + 0.6 + 0.8 + 1).

In this case, the value is calculated as follows

$$\frac{[(77*0.2)+(83*0.4)+(75*0.6)+(88*0.8)+(66*1)]+249+262+259.6\text{ (due to the other 3 variables)}}{2400},$$

which in turn corresponds to the value 1.2507 ($\in [0, 3]$).

Herewith, $(77 * 0{,}2) + (83 * 0{,}4) + (75 * 0{,}6) + (88 * 0{,}8) + (66 * 1)$ comes from the variable $x5_2$, as an example, where 77, 83, 75, 88, 66 stands for the summation of each row for $x5_2$ with different weight values, 0.2, 0.4, 0.6, 0.8, 1, respectively.

Again, rounding the value 1.2507 for practical purposes yields the ***R-value 2*** = **1.3**. Thus, for the given dataset of *MQ2* with its particular distribution of numbers (data), the versions or alternatives of *R*-value 1 and *R*-value 2 gave the *same* result after rounding.

R-value 3 is also created based on the *Score Matrix 2* and delivers the value 0.694, rounded to the already relatively low *R-value 3 = 0.7*.

However, it was preferred to have full, comprehensive and "positive" $R\text{-}values$, which therefore are bigger than 1. As a result, the $R\text{-}value\ 2$ of **1.3** was selected as $the\ Reference\ Value$. This reference value was introduced to the company while it was asked for the R_{tjkn} values to be used as a reference point for the related values.

Now, because of the choice of increasing weights for the used data from *MQ2* here, *R-value 2* could be called an *"Appreciation Factor"* or *"Positivity Factor"* with regard to the 4 regarded input variables - according to the estimated model, parameters - of workforce satisfaction. Due to its advantages in terms of its included and assessed specificity, appreciation and positivity, the R-value 2 also compensates for the disadvantage of a slightly lower amount of about 1.2507 compared to the R-value of about 1.264. Different R-values dependent on time horizon, products, work shifts and workforce groups are given in *Appendix G*.

3.4.3. MODM Results

As discussed, the case study was conducted in Iran at *Beshel Motor Industry.* In this regard, the questionnaires were completed and then data collection was done in order to set the main parameters of Section 3 on MODM by the help of MARS and Scoring Matrix method as well as the innovations and tables. *Four weeks* (June 1, 2021 - June 28, 2021) were considered as the time horizon as well as the production department of *Cylinder engine block* for *PRIDE* (see Figure 4).

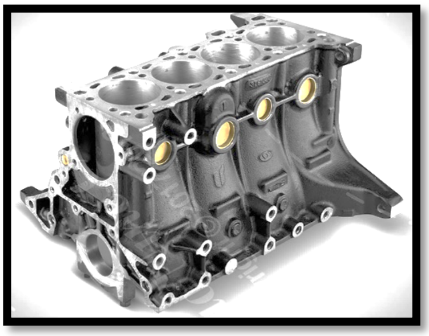


Figure 4. Cylinder engine block for PRIDE.

In this department, *3 types of products* from *PRIDE* family are produced (Pride, Tiba, Saina). There are just slight differences between these three types.

Now, CONOPT solver/GAMS software is used to find the solution to the final MINLP model. The codes are given in *Appendix C*. Table 25 shows the obtained computational results where the ideal values of the 1st and 2nd objective functions are $b1$ and $b2$. Moreover, $Z1$, $Z2$ and WGP represent the optimal values of the 1st, 2nd and WGP objective functions, respectively. The *Reliability Factor* has been unfolded by 4 different R variables: $R1$, $R2$, $R3$ and $R4$, as represented in Eqn. (56). (The reverse of this unfolding process, the so-called *"leave-1-out"* method, will be useful a little later.) Moreover, 5 workforce groups related to their skill levels and based on their working years (including supervisor / production manager, foreman and operators) are defined. The Table 25 shows obtained results for *R* value.

Table 25. Obtained computational results.

	***b*1**	***b*2**	***Z*1**	***Z*2**	***WGP***
R	8920482	171.744	12049000	173.701	0.181
***R*1**	5211854	171.744	8413986	173.977	0.314
***R*2**	5074944	171.744	12132000	173.743	0.701
***R*3**	5033306	171.744	8833001	174.398	0.385
***R*4**	8050627	171.744	10327240	171.338	0.145

As can be seen, *b*2 remains fixed over different *R*-values while *Z*2 reflects small changes which imply the stability and robustness of the APP system. In other words, the reliability function just changed slightly over different *R*-values. To learn more about the corresponding aggregate production plan, there are both normal and overtime production and no outsourcing is incorporated into the optimal APP system (Tables 26-27). More details about company's data are given in *Appendix B.*

Table 26. In-house production amount.

y_{tnj}^{PR}	***j*=1**	***j*=2**	***j*=3**
***t*=1** and ***j*=1**	111	111	111
***t*=1** and ***j*=2**	111	111	111
***t*=1** and ***j*=3**	108	108	108
***t*=2** and ***j*=1**	111	111	111
***t*=2** and ***j*=2**	111	111	111
***t*=2** and ***j*=3**	108	108	108
***t*=3** and ***j*=1**	111	111	111
***t*=3** and ***j*=2**	111	111	111
***t*=3** and ***j*=3**	108	108	108
***t*=4** and ***j*=1**	111	111	111
***t*=4** and ***j*=2**	111	111	111
***t*=4** and ***j*=3**	108	108	108

Table 27. Amount of products delivered to customers.

x_{tn}	***n*=1**	***n*=2**	***n*=3**
***t*=1**	700	944	672
***t*=2**	332	333	323
***t*=3**	13828	333	13893
***t*=4**	332	13339	323

Therefore, the obtained optimal policy is recommended to managers to be implemented in the company, as it thoroughly takes into account human resource management along with APP, ensuring a balanced approach that enhances operational efficiency while addressing workforce considerations.

CHAPTER 4
Discussion and Conclusion

4. Discussion and Conclusion

4.1. Sensitivity Analysis

Here, the sensitivity analysis of the key parameters of the mathematical model is discussed. The purpose of sensitivity analysis is to determine the effect of key parameters on the optimal value of the objective functions. To do so, R, TRC and m are taken into account to conduct the sensitivity analysis. Table 28 shows the sensitivity analysis results of R when this parameter is active and when it is not active in the systems.

Table 28. Sensitivity analysis of R.

	*b*1	*b*2	*Z*1	*Z*2	*WGP*
With *R*	8920482	171.744	12049000	173.701	0.181
Without *R*	13680460	180.000	1.37E+07	180.000	0.000

Furthermore, when the R factor is removed from the system totally (i.e., $0 * R$), the system gets $b2$=180. So, the maximum value (the worst value will be 180 which is around 4.8% more). The point is now to justify that -4.8% change in $Z2$ and finally the APP system. Therefore, considering human factors can lead to a 4.8% improvement in the stability of the system which is not ignorable definitely. Additionally, it can be seen an increase in the cost amount once the R factor is removed from the system. The cost got increased from $b1$ = \$8,920,481.827 to $b1$= \$13,680,460. So, the change in cost is 53.3%.

Here, the *"leave-1-out"* method from statistics, statistical and machine learning is applied in a very new frame and context. In fact, in order to view *R*=0 from a different, limit-value perspective, "leave-1-out" is applied 4-times, one after the other, on the 4 variables in the definition of the *R*-value and on the way towards *R*=0.

Figure 5 displays the impact of R on $b1$ & $Z1$ and $b2$ & $Z2$ explicitly.

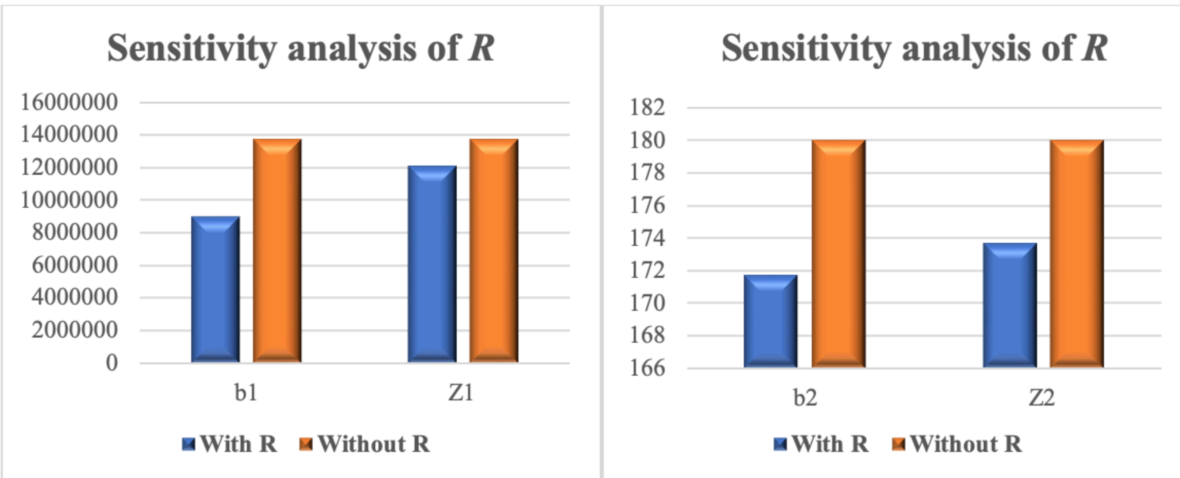


Figure 5. Sensitivity analysis of R: $b1$ & $Z1$ and $b2$ & $Z2$.

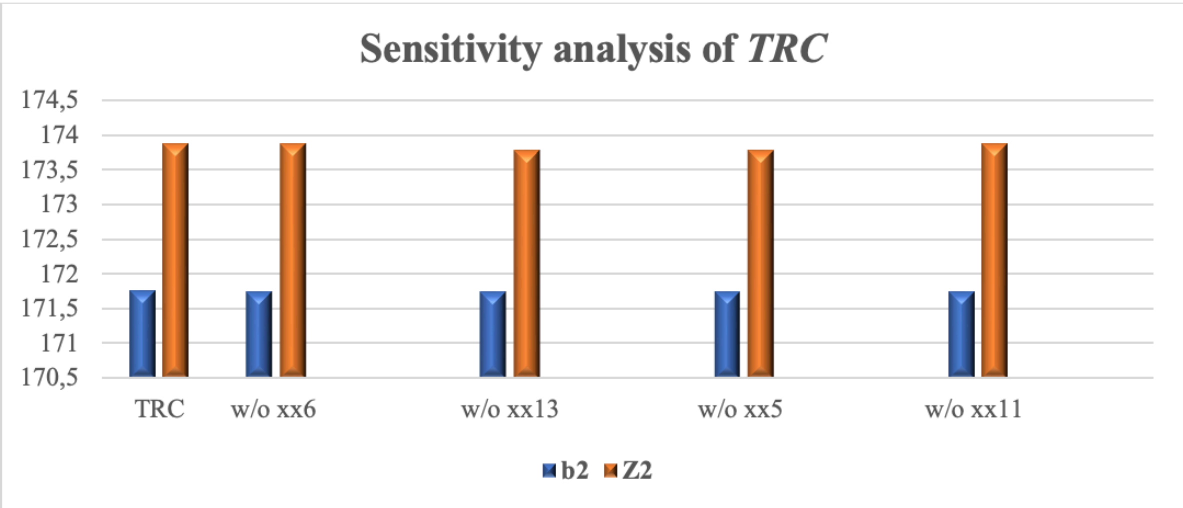
Table 29 illustrates the sensitivity analysis of *TRC*, in which the number of effective elements in its formula changes. In other words, the contribution of each element will be excluded to see how the model behaves.

Table 29. Sensitivity analysis of *TRC*.

	$b1$	$b2$	$Z1$	$Z2$	WGP
TRC	8866190	171.765	1.19E+07	173.88	0.243
w/o $xx6$	8919031	171.744	1.25E+07	173.88	0.283
w/o $xx13$	8866190	171.744	1.06E+07	173.79	0.137
w/o $xx5$	8866190	171.744	1.06E+07	173.79	0.137
w/o $xx11$	8866190	171.744	1.13E+07	173.88	0.199

Here, it can be seen that the most effective parameter among these 4 is *General Skills* ($x6$) according to given dataset. Detailed information about skill groups is given in the introduction part of the $MQ1$.

Figure 6 displays the impact of TRC on $b1$& $Z1$ and $b2$ & $Z2$ explicitly.



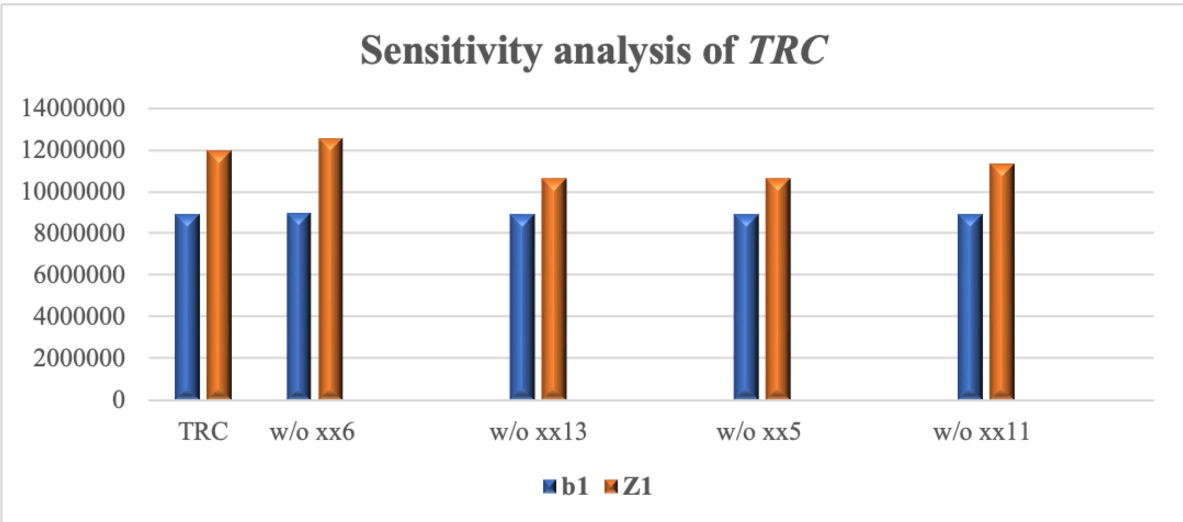


Figure 6. Sensitivity analysis of *TRC*: $b1$&Z1 and $b2$&$Z2$.

As with *TRC*, the sensitivity analysis for the parameter *m* is now shown in Table 30.

Table 30. Sensitivity analysis of m.

	$b1$	$b2$	$Z1$	$Z2$	WGP
m_{nj}	8866190	171.744	2.21E+07	173.866	0.755
w/o $xx9$	9156398	171.744	1.34E+07	173.699	0.236
$xx15$	8674635	171.744	2.44E+07	173.905	0.911
$xx27$	8863982	171.744	2.16E+07	173.88	0.722
m_0	10202570	171.859	1.18E+07	172.279	0.080

Here, the most effective parameter among these 3 is *Space*, *Lighting* and *Ventilation*. Its absence can lead to an increase in the total cost of the system, from \$8,866,190 to \$9,156,398. Furthermore, when man-hour-related parameters are removed from the system, only running it with m_0, the total cost of the system got increased by 15%, from \$8,866,189.5486 to \$10,202,570.

Figure 7 displays the impact of m on $b1$ & $Z1$ and $b2$ & $Z2$ explicitly.

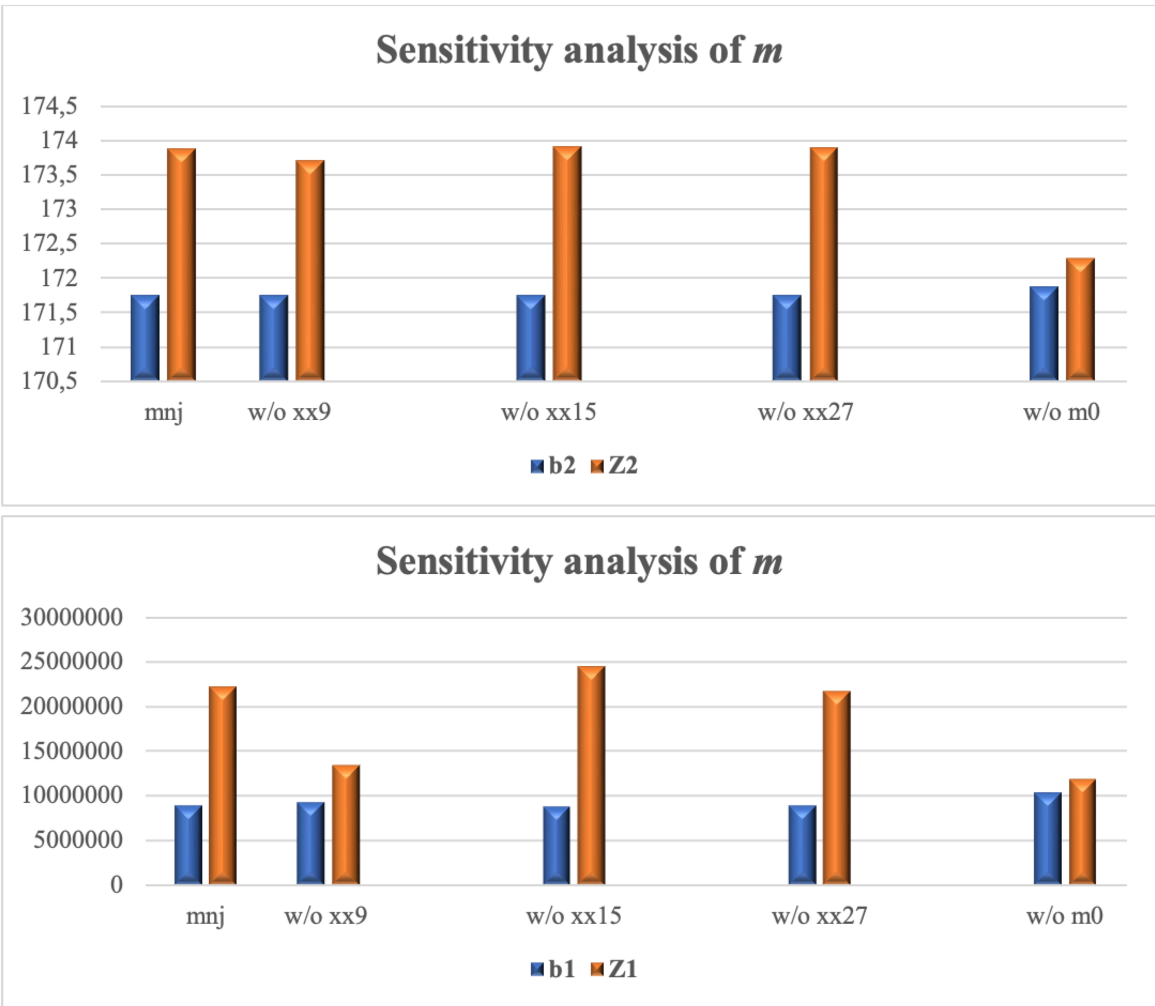


Figure 7. Sensitivity analysis of m: $b1$ & $Z1$ and $b2$ & $Z2$.

4.2. Practical Implications and Managerial Insights

The findings and methodologies developed in this study provide several significant practical implications and managerial insights that address contemporary challenges in APP with a focus on human factors, sustainability, and reliability.

- **Integration of Human Factors in Decision-Making**

This research emphasizes the critical role of human-related variables in APP. By systematically including human factors (e.g., skill sets and satisfaction factors), the proposed framework enables managers to optimize production systems while simultaneously improving employee well-being both mentally and physically. These important objectives ensure a balance between operational efficiency and workforce sustainability, providing a resilient and adaptable production environment.

Managers can utilise the developed Matrix Questionnaires (*MQ1* and *MQ2*) as diagnostic tools to assess the impact of various workforce-related factors on productivity and system reliability. These questionnaires are not only highlight areas for improvement but also provide actionable insights into training and workforce allocation strategies in conjunction with organisational needs.

- **Enhanced Reliability through Learning and Adaptability**

The incorporation of reliability as one of the core objectives, modeled through learning rates, fatigue, and forgetting curves, equips managers with an exact understanding of workforce dynamics. This approach helps to identify optimal workforce configurations, work shift setting, and training systems that minimise production disruptions while maximising workforce stability and satisfaction. By aligning production goals with workforce capabilities, organizations can achieve higher reliability levels, essential for JIT systems, and finally customer satisfaction.

- **Cost Optimization in Complex Production Scenarios**

The bi-objective mathematical model developed in this study offers a robust tool for balancing cost minimisation with system stability. By leveraging WGP and Fuzzy Programming techniques, managers can make informed decisions under uncertain conditions, such as fluctuating demand and cost as well as workforce variability. The model's flexibility allows its application across diverse industries, including automotive, where production environments are characterized by high complexity and uncertainty.

- **Data-Driven Insights with Advanced Analytical Tools**

The use of MARS for analyzing workforce and production data enables a precise identification of critical factors influencing production outcomes. This data-driven approach empowers managers to focus resources on high-impact areas, leading to improved decision-making and strategic planning. Furthermore, the derived scoring matrixes provide a practical reference for prioritizing interventions and aligning operational strategies with organisational objectives.

- **Contribution to Sustainable Development Goals**

By integrating human factors and emphasizing workforce satisfaction, this research contributes to broader sustainability goals. Organisations adopting the proposed APP framework can expect not only economic benefits but also enhanced social sustainability. Higher job satisfaction and employee well-being lead to lower (employee) turnover rates, higher engagement and long-term operational stability and are consistent with corporate social responsibility (CSR) goals.

- **Managerial Implications**

The following managerial insights are drawn for the studied case problem:

1. ***Proactive Workforce Management:*** The insights from this research highlight the necessity for managers to invest in workforce development programs and adopt optimal APP strategies.
2. ***Strategic Use of Technology:*** By integrating advanced analytics and decision-support tools, managers can optimise production plans with a comprehensive understanding of human and technical factors.
3. ***Balancing Objectives:*** The bi-objective approach ensures that cost efficiency does not come at the expense of workforce satisfaction and system reliability.
4. ***Flexibility in Uncertainty:*** The model's robustness against demand fluctuations provides managers with confidence to handle uncertain market conditions effectively.

All in all, the proposed framework and findings from this study serve as a roadmap for managers seeking to enhance production planning through the integration of human-centric and sustainable practices, ensuring a competitive edge in today's dynamic industrial landscape.

4.3. Limitations and Outlook of the Research

Despite the comprehensive scope and methodological rigor of this study, several limitations must be acknowledged. These limitations provide opportunities for improvement and set the stage for future research in APP with human factors:

I. Data Availability and Quality

The reliability of the findings depends on the quality and comprehensiveness of the data collected via Matrix Questionnaires (*MQ1* and *MQ2*). While the study leveraged data from a real-life case in the automotive industry, potential biases in respondents' feedback, incomplete responses, or variability in operational contexts could affect the generalisability of the results. Furthermore, reliance on a single industry case may limit the broader applicability of the results to other manufacturing sectors or regions with different workforce dynamics and production systems.

II. Complexity of the Mathematical Model

The bi-objective MINLP model presented in this research is computationally intensive, particularly for large-scale problems involving numerous variables, products, and time periods.

While robust optimisation techniques and solver tools like CONOPT in GAMS were employed, scalability remains a challenge for real-time or highly dynamic production environments.

III. Simplified Assumptions

Certain assumptions, such as the uniformity of workforce learning rates, fatigue coefficients, and forgetting curves, may not fully capture the diversity of individual worker behavior or varying industrial practices. Additionally, the treatment of demand uncertainty using triangular fuzzy numbers, while effective, may not reflect more complex probabilistic distributions observed in some industries.

IV. Limited Exploration of Interdependencies

The interactions between human factors and technical aspects of production (e.g., machinery efficiency, maintenance schedules) were not extensively explored. This simplification might overlook synergistic or conflicting effects that could influence the APP outcomes.

V. Context-Specific Constraints

The case study is specific to the automotive industry and its unique requirements, such as multi-product production and JIT policies. This specificity may limit the direct applicability of the framework to industries with distinct operational characteristics, such as batch production or process manufacturing.

Now, building on the contributions and limitations of this study, several promising avenues for future research emerge:

i. Expanding Industry Applications

Future studies can apply the proposed APP framework to diverse industries, such as pharmaceuticals, electronics, and food processing, to validate its versatility. Comparative analyses across sectors could uncover industry-specific variations and lead to the development of efficient optimisation strategies.

ii. Integration with Emerging Technologies

The integration of advanced technologies, such as the IoT, digital twins, and AI, could enhance the framework's real-time decision-making capabilities. For instance, IoT-enabled sensors can provide real-time data on workforce fatigue or equipment performance, while AI algorithms are able to dynamically adjust production plans based on shifting demand patterns.

iii. Exploration of Nonlinear and Dynamic Interactions

The nonlinear relationships and dynamic interactions between human factors and other production elements, such as equipment reliability and supply chain constraints, may be investigated. Incorporating these interdependencies into the mathematical model can yield more comprehensive and realistic optimization results.

iv. Enhanced Demand Uncertainty Modeling

To better represent the complexity of demand patterns, future studies could explore advanced uncertainty modeling techniques, such as stochastic processes, Bayesian networks, or machine learning-based demand forecasting. This would provide a more robust foundation for APP in volatile market conditions.

v. Scalability and Computational Efficiency

Efforts to develop more scalable algorithms or employ parallel computing techniques can make the framework more suitable for large-scale industrial applications. Incorporating heuristic or metaheuristic approaches, (e.g., genetic algorithms or simulated annealing) addresses computational challenges while maintaining solution quality.

vi. Broader Workforce Considerations

Further human factors, including psychological and social dimensions, such as worker motivation, team dynamics, and leadership influences, can be studied and incorporated into the model to address both operational and organisational aspects of APP.

vii. Sustainability and Circularity Integration

Since the model studied in this project is based on the *JIT production system*, it offers benefits like cost savings and increased efficiency, while it also has drawbacks, especially concerning the environmental cost such as increased carbon footprint, strain on local ecosystems, waste accumulation, and pressure on natural resources. Expanding the model to incorporate circular economy principles and environmental sustainability metrics, such as environmental pollution minimisation, waste reduction or energy efficiency maximisation as the *3rd objective function*, could enhance its relevance in the context of global sustainability goals. Therefore, the following strategies are recommended to the company:

- Encouraging the use of renewable energy for production and transportation (delivery),
- Implementing sustainable sourcing practices (e.g., sustainable supplier selection),
- Considering reusable or minimal packaging in shipments,
- Incorporating circular economy principles into production to reduce waste and environmental impact.

viii. Real-Time Decision-Making Systems

Developing decision-support systems that integrate the proposed APP framework into real-time, interactive platforms can improve its usability for practitioners. Such systems allow managers to visualize trade-offs, simulate scenarios, and make adaptive decisions based on real-time data.

The limitations identified in this study underscore the complexity of integrating human factors into APP while addressing reliability, cost efficiency, and uncertainty. However, the insights gained and the proposed avenues for future research lay a strong foundation for advancing the field. By addressing these challenges, future studies can contribute to the development of more adaptive, human-centric, and sustainable APP systems across industries.

4.4. Summaries

The aforementioned research findings provide valuable insights into the relationship between human factors, sustainability and reliability for an aggregate production plan. The research aimed to achieve several objectives and obtain a framework for decision makers. The findings have revealed the impact of several human factors for the production system. The results emphasise how crucial it is to have deeper understanding of needs of people (workers) during decision-making processes besides the systems' needs. The obtained framework can be used by managers or practitioners as a tool in order to have a broader perspective when it comes to make decisions under uncertain environments, not only with uncertainty in demand but also with human related scenarios.

The impact of working conditions on the system's overall cost has been demonstrated. An ideal work environment includes things like the capacity to share experiences, relevant organisational structures, adequate lighting, enough room for operations, and adequate ventilation. In their absence, the system's optimal man-hour requirements are increased, which raises the system's overall cost significantly.

Human factors have been found to be quite important in the process of maintaining a reliable system. Managers should prioritise paying attention into their relationships with the employees by giving enough respect for esteem needs, securing the justice with their staff, providing trustful working environment and promising the opportunities for better career plans for the future. It gives employees satisfaction with their work lives and consequently creates long-term relationship status with their employers. Having qualified staff for the long term can result in better customer connections, and raising satisfaction levels can boost market competitiveness as well. Herewith, it is crucial to say that general system at work, job satisfaction factors and human resource management operations play a big role to achieve reliable systems.

Furthermore, from the obtained results, it can be seen that general skills have the most impact on the system. General skills have bigger role than technical skills or trainings/education of people at the work places. General skills are mainly defined as *Team working*, *Problem solving*, *Analytical thinking*, *Decision*, while technical skills are considered as *Working with tools and technology*, *Mechanical*, *business fundamentals*, and *common skills* are *Basic mathematics*, *Writing and reading*, *Customer orientation*, *Professionalism*, *Adaptability*. As it can be seen, human skills (general skills) are more important when it comes to sustainability of the system. General skills efficiently contribute into the cost efficiency goals of the system. From the goal perspective of this study, it is one of the good results that general abilities of being human help people more than any knowledge in their work lives. It reflects the importance of the human beings for the systems. This again illustrates that being able to solve problems or make decisions are more important for having cost-efficient system than knowing how to use a machine. Furthermore, having good analytical skills also can help workers to dive deeply into the source of any problem. This will lead to less complexities in current situation and consequently less probability of having any extra cost in the system. The results of this study for *TRC* factor already support this point as well.

The methodology part of the study is also important as it brings another perspective when it comes to analyse complex systems. Plenty factors have been related with the problem but by the help of the MARS, importance level of each factor in the system have been efficiently determined. This will help managers to save their time and effort while getting better understanding of the system as MARS ensures *transparency* and *interpretability* with its steps, while other machine learning and artificial intelligence algorithms and heuristics often appear as “black boxes”.

Additionally, this doctoral thesis is not directly about *human resource management* in general, but according to an aggregate production plan, with the aim of minimising the total costs and maximizing the *human factor* supported (*APP-HF*) and committed total reliability. Due to this specific nature of this study, it is not emphasized or generalized about human resource operations and optimal decisions about or from them, but it is a must to say that better management in workforce leads to significant advancements of a company.

However, it should also be emphasized that this doctoral study looks further into the future - even in the sense of a research agenda - than 2 main objective functions together with their numerous, also HR-related components and properties suggest. In particular, this doctoral study thesis as a whole is much more than the mere sum of its parts. In fact, the study points to the future development of integrated HR-committed and -supported *Graphical User Interface*s (*GUI*s), which aim to greatly reduce the workload of the decision makers or managers, not least the HR operations managers. These GUIs will then ideally contain, offer in a pleasant way and make accessible a graphical, interactive and playful use of the MARS and CMARS, RMARS and RCMARS software tools, of the methods and techniques of Sensitivity Analyses and Simulations, of the derivation, presentation and implementation of Managerial Conclusions.

In this PhD. study, 8 research questions have been answered.

***RQ1** – What (which variables) are “Human Factors” with a possible impact on Aggregate Production Planning?*

In order to systematically collect the information for the possible impactful Human Factors, 2 questionnaires (*MQ1* & *MQ2*) have been created for this study. Herewith, “human factors” cover a very broad and detailed range of mental and behavioural, individual and collective expressions and responses of human life, as far as they are related with a company’s activities, whether they are manual or operational, decisional or managerial. Human factors are taken into account richly, whether their expressions and responses are of a more qualitative or a more quantitative nature, and instruments, methods and tools from mathematics and statistics, natural and human sciences are employed in order to evaluate them for further modelling and eventually decision making. This includes the creation and introduction of human-factor-based parameters as well.

***RQ2** – What input and output variables are crucial for cost-efficient and reliable decision-making?*

For this study, 9 criteria (output variables) and 50 related factors (input variables) have been primarily used in the questionnaires. After data applications, 11 of the factors were selected as the most important ones for each related criterion. Table 31 shows the final important input variables with their codes and explanations. The more information about criteria can be found on page 62.

Table 31. Final important factors for the whole system.

Code	Description
$x5_1$	Needed technical or professional skills
$x6_1$	Needed general skills
$x11_1$	Effects of place of work
$x13_1$	Importance of training and education
$x5_2$	Satisfaction with respect for esteem needs by managers
$x9_2$	Space, lighting and ventilation
$x15_2$	Professional atmosphere / availability to share experience
$x21$	Trust in director and company generally
$x27$	Forms of work organization
$x30$	Treating employees fairly and equally
$x35$	Career opportunities

***RQ3** – What are the intercultural and multidisciplinary constraints and obstacles for data collection?*

To have successfully collaborated data on a research project with a country like Iran in times of great international crisis and war is a remarkable shared achievement for all those involved. Collaborating researchers came from Poland, Turkey, Iran, Germany and Singapore and were geographically separated. In addition, the project work overlapped with the COVID-19 pandemic.

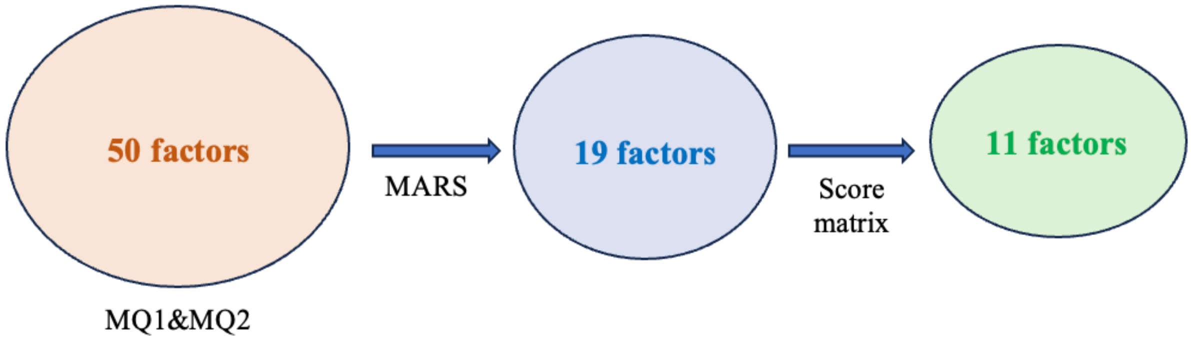
The research project itself brought together scientists and practitioners from management, mechanical engineering, human sciences, mathematics, statistics, industrial engineering and computer science and can therefore be viewed as extremely interdisciplinary. During this doctoral research project, it had to be anticipated and incorporated, that questions had to be posed very carefully, taking into account cultural and societal hesitations, forbids and even taboos.

At the same time, the scientific study required sufficient certainty and assurance on the part of the answers or data, so that the overall communication and learning process with its several stages and the selection of strong scientific instruments always had to be both sensitive, even empathetic, and reinforcing.

RQ4 – How to process data obtained from the MQs?

In this study, MARS application has been used for the analysis of obtained data due to its superiority. Throughout MARS application, 19 of 50 factors were selected as important ones (more important). After MARS, by the use of Score Matrixes, the most impactful factors have been determined. Figure 8 shows the change in the number of factors and the steps of the process for the data analysis.

Figure 8. The change in the number of factors.



RQ5 – What is the relation between HFs and reliability and cost criterion?

The relation between human factors and reliability of the system, as well as cost aspects, has been mainly illustrated through *m*-, *TRC*- and *R*-factors. Based on MARS results, 3 parameters, namely manhour related - *m*, training-cost related - *TRC*, and reliability factor - *R*, have been created, been included into APP model using objective functions and constraints, and their importance have been proven for cost efficiency and reliability of the system. The related formulas of the variables are as follows:

$m_{nj} = m0 + \left(xx9_2\; m1_{nj} + xx15_2\; m2_{nj} + xx27\; m3_{nj}\right),$
$TRC_{kn} = xx6_1\; TRC0_{kn} + (xx13_1 + xx5_1)\; TRC1_{kn} + xx11_1\; TRC_{2kn},$
$R_{tjkn} = x5_2 + x21 + x30 + x35.$

The explanation of the formulas is on pages 68-70.

RQ6 – How to include and cope with uncertainty in APP?

Firstly, the powerful tool of MARS method has been employed to cope with uncertainty from the perspective of statistics on uncertainty in the data. This scientific method was enabled and supported by regular contact with the experts in the partner company. Secondly, the powerful methodology of Fuzzy Logic Programming has been used to deal with the uncertainty in demand. Thirdly, sensitivity analysis methodologies have been created and applied in order to simulate various parametric scenarios which are uncertain in nature. All of these 3 scientific strategies strongly reduced the otherwise high impact of uncertainty on both modelling and decision making.

RQ7 *– What is the impact of HF on quality/efficiency of APP and its results?*

Several human factors have been included into this decision-making framework, and throughout Sensitivity Analysis, the impact of their absence/presence on cost and reliability of the system has been proven. The most important results are shown in Table 32 and more information can be found on the pages 76-78.

Table 32. Impact of absence/presence on cost and reliability of the APP system.

	***b*1**	***b*2**	***Z*1**	***Z*2**	***WGP***
With *R*	8920482	171.744	12049000	173.701	0.181
Without *R*	13680460	180.000	1.37E+07	180.000	0.000

RQ8 *– What are the managerial implications of APP?*

In order to assist managers in making decisions processes, this study has examined the elements of *human-related* scenarios in production plans. The new framework will help practitioners who wish to make their system more human-friendly by a more comprehensive approach.

The newly invented managerial instruments, methods and tools include:

- ***Human-Factor based APP*** (as an integrated managerial model and decision support system): *achieved,*
- ***MARS models in APP***: *achieved,*
- ***Reliability*** in Human-Factor based APP (as robust measure and criterion): achieved,
- **Score Matrices** (including generalized correlation coefficients and substitution effects): *achieved,*
- ***R-values, M-values, and TRC-values***: *achieved,*
- ***Coping with "internal", collective and ideal data***: *achieved,*
- ***Individualizing*** "derived (collective) data (or parameter values)" (through Workforce Groups): *achieved,*
- ***Experimental Statistical Design*** (including the previous items): *achieved,*
- ***Sensitivity Analyses*** (also for "testing"): *achieved*,
- ***Simulation*** (based on MARS models and sensitivity analyses): *achieved* (can be worked out in future),
- ***APP Game Theory*** (every achieved matrix or table corresponds to a game; altogether, an APP Game): *achieved* (can be worked out in future),
- ***Graphical User Interfaces*** (*GUIs*) (for sensitivity analyses, simulation and for the whole APP): *achieved* (can be worked out in future),
- ***Environment- and Nature-Co-based APP*** (from Humans to Living Being (Creatures)): *achieved* (can be worked out in future).

Appendix A. Questionnaires

1. MATRIX QUESTIONNAIRE REGARDING GENERAL SYSTEM

Please note:

This questionnaire has been created to observe the General System at the workplaces.

You are expected to assign the appropriate score for each cell connecting each row to each column. For each cell in the table, I kindly ask you for one number as your assessment of an average value for the importance of each factor to each criterion with respect to employees in your company.

Please note that you should do your assessments on the behalf of your employees by considering their situations in your company.

This matrix can be filled out by a set of managers in different departments (production, quality control, quality assurance, maintenance, etc.), or by a scholar (expert) who knows the company very well. I asked the respondent for the criteria and sub-criteria, namely for matrix cells in the "crosshairs" between sub-factors (or levels) and sub-criteria, to pick values in the interval [0, 10], where 0 means the lowest while 10 represent the highest value.

Specific skills: Technical/professional skills needed in all proficiencies of the company.
(**Examples:** Working with tools and technology, Mechanical, Business fundamentals)

***Common skills*:** Skills in common among employees of one department.
(**Examples:** Basic mathematics, Writing and reading, Customer orientation, Professionalism, Adaptability)

***General skills*:** All skills in common for all divisions at the departments of the company.
(**Examples:** Team working, Problem solving, Analytical thinking, Decision making)

- **MQ1**

Criteria ***1. Matrix Questionnaire*** **Factors**	**Overall Fulfilment of Duties** *Scale*: 0-10 (e.g., 0.1, 5, 9.8)	**Quality of Contribution** (related to production) *Scale*: 0-10 (e.g., 0.1, 5, 9.8)	**Production Level** (in unit time) *Scale*: 0-10 (e.g., 0.1, 5, 9.8)	**Flexibility at Work** *Scale*: 0-10 (e.g., 0.1, 5, 9.8)
Importance of Age: 16-30, 31-50, 51-65, 66-80				
Importance of Gender: Male, Female				
Importance of Marital tatus: Single, Married				
Importance of Educational Level: Technical, High School, University, Master, PhD				
Needed "Specific (technical or professional) Skills" Satisfaction Level in the company: Between company's demand and employees' fulfilment				
Needed "General Skills" Satisfaction Level in the company: Between company's demand and employees' fulfilment				

Criteria ***1. Matrix Questionnaire*** **Factors**	**Overall Fulfilment of Duties** *Scale*: 0-10 (e.g., 0.1, 5, 9.8)	**Quality of Contribution** (related to production) *Scale*: 0-10 (e.g., 0.1, 5, 9.8)	**Production Level** (in unit time) *Scale*: 0-10 (e.g., 0.1, 5, 9.8)	**Flexibility at Work** *Scale*: 0-10 (e.g., 0.1, 5, 9.8)
Needed "Common Skills" Satisfaction Level in the company: Between company's demand and employees' fulfilment				
Needed "Qualifications" Satisfaction Level in the company: Between company's demand and employees' fulfilment				
Effect of Workplace Distance to home: Far Away, Near				
Effects of Employment Type: Full-time, Part-time, Fixed term contract, Piece rate				
Effects of Place of Work: Non-Stationary, Stationary				
Effects of Shift Work: Night shift, Early morning shift, Rotating shift				
Importance of Training and Education: Long-term train, Short-term train				

Criteria ***1. Matrix Questionnaire*** **Factors**	**Overall Fulfilment of Duties** *Scale*: 0-10 (e.g., 0.1, 5, 9.8)	**Quality of Contribution** (related to production) *Scale*: 0-10 (e.g., 0.1, 5, 9.8)	**Production Level** (in unit time) *Scale*: 0-10 (e.g., 0.1, 5, 9.8)	**Flexibility at Work** *Scale*: 0-10 (e.g., 0.1, 5, 9.8)
Effect of Employees' Personalities: Anxiety, Extroversion, Religiosity, Depression				
Importance of Level of Employees' Salary: Low, Average, High				

2. MATRIX QUESTIONNAIRE
REGARDING
WORKFORCE SATISFACTION

Please note:

This questionnaire has been created to observe the Workforce Satisfaction with respect to the working environment of the workplaces.

You are expected to assign the appropriate scores for each cells at each level in rows (representing values of different levels) with connecting each column. For each cell in the table, I kindly ask you for one number as your assessment of how the each factor contributes to each criterion (at different levels).

Please note that you should do your assessments on the behalf of your employees by considering their situations in your company.

Here is the matrix that should be filled out by supervisors, foremen or expert scholars who know opinions / feelings of the employees very well in your company. I asked the respondent for the criteria and sub-criteria, namely for matrix cells in the "crosshairs" between sub-factors (or levels) and sub-criteria, to pick values in the interval [0, 10], where 0 means the lowest while 10 represents the highest value.

- **MQ2**

Criteria ***2. Matrix Questionnaire*** **Factors**	**Job Satisfaction** *Scale*: 0-10 (e.g., 0.1, 5, 9.8)	**Health at Risk** *Scale*: 0-10 (e.g., 0.1, 5, 9.8)	**Relationship with co-workers** *Scale*: 0-10 (e.g., 0.1, 5, 9.8)	**System at Work** *Scale*: 0-10 (e.g., 0.1, 5, 9.8)	**Human Resource Management Operations** *Scale*: 0-10 (e.g., 0.1, 5, 9.8)
Satisfaction with sense of achievements					
Satisfaction by being well-matched with working position					
Sense of integration / belonging into the working process / community					
Satisfaction with the trust felt by the managers in general					

Criteria ***2. Matrix Questionnaire*** **Factors**	**Job Satisfaction** *Scale*: 0-10 (e.g., 0.1, 5, 9.8)	**Health at Risk** *Scale*: 0-10 (e.g., 0.1, 5, 9.8)	**Relationship with co-workers** *Scale*: 0-10 (e.g., 0.1, 5, 9.8)	**System at Work** *Scale*: 0-10 (e.g., 0.1, 5, 9.8)	**Human Resource Management Operations** *Scale*: 0-10 (e.g., 0.1, 5, 9.8)
Satisfaction with respect for esteem needs by managers					
Satisfaction with current maintainance of the workplace					
Satisfaction with access to materials and equipments					
Occupational health and safety					
Space, lighting and ventilation					
Ergonomic / physical arrangement of work area					
Hygiene and sanitary at the workplace					
Physical health and mental health first aid					
Private health and accident insurance benefits					
Social security benefits					
Professional atmosphere / availability to share experiences					
Communication channels opportunities in the company					
Interpersonal relationships between employees					
Communication style of managers with employees					
Reasonable expectations of manager					

Criteria ***2. Matrix Questionnaire*** **Factors**	**Job Satisfaction** *Scale*: 0-10 (e.g., 0.1, 5, 9.8)	**Health at Risk** *Scale*: 0-10 (e.g., 0.1, 5, 9.8)	**Relationship with co-workers** *Scale*: 0-10 (e.g., 0.1, 5, 9.8)	**System at Work** *Scale*: 0-10 (e.g., 0.1, 5, 9.8)	**Human Resource Management Operations** *Scale*: 0-10 (e.g., 0.1, 5, 9.8)
Respect / consideration for expectations of workers by managers					
Trust in director and company generally					
Satisfaction with working time					
Courtesy for private issues by manager					
Company attitudes during crisis time					
Satisfaction with amount of payment					
Flexible and home working					
Forms of work organization					
Punishment and reward system					
Strategyies to resolve conflicts among employees					
Treating employees fairly and equally					
Selection of the right persons for the right positions					
Proper personality analysis of workers in general					
Recreational facilities					
Traning programmes to enhance work performance					
Career opportunities					

Appendix B. MODM Data

Table B1. Initial inventory amount of final products.

$I_{0,n}$	**Value**
$n = 1$	230
$n = 2$	750
$n = 3$	150

Table B2. Minimal experience of a worker from workforce group *k* to work in department *n* (in terms of production).

LI_{kn}	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$
$n = 1$	0	1	2	4	6
$n = 2$	0	1	2	4	6
$n = 3$	0	1	2	4	6

Table B3. Maximum efficiency of a worker from workforce group *k* to work in department *n* in work shift *j* (depending on production rate).

KI_{knj}	$j = 1$	$j = 2$	$j = 3$
$k = 1, n = 1$	8	8	9
$k = 1, n = 2$	8	9	9
$k = 1, n = 3$	10	10	8
$k = 2, n = 1$	9	9	10
$k = 2, n = 2$	9	11	9
$k = 2, n = 3$	12	11	9
$k = 3, n = 1$	11	11	13
$k = 3, n = 2$	12	11	12
$k = 3, n = 3$	14	12	13
$k = 4, n = 1$	15	15	16
$k = 4, n = 2$	18	13	18
$k = 4, n = 3$	21	15	20
$k = 5, n = 1$	24	24	23
$k = 5, n = 2$	26	24	22
$k = 5, n = 3$	26	25	24

Table B4. Learning rate of a worker from workforce group *k* in department *n* at the beginning of work.

LE_{kn}	$n = 1$	$n = 2$	$n = 3$
$k = 1$	0.3	0.4	0.4
$k = 2$	0.4	0.4	0.5
$k = 3$	0.5	0.4	0.6
$k = 4$	0.5	0.5	0.5
$k = 5$	0.6	0.7	0.7

Table B5. Forgetting rate of a worker from workforce group *k* in department *n* in period *t* work shift. j.

LF_{kntj}	$t = 1,2,3,4$		
	$j = 1$	$j = 2$	$j = 3$
$k = 1, n = 1$	0.05	0.06	0.08
$k = 1, n = 2$	0.1	0.1	0.1
$k = 1, n = 3$	0.07	0.09	0.09
$k = 2, n = 1$	0.04	0.05	0.06
$k = 2, n = 2$	0.09	0.08	0.08
$k = 2, n = 3$	0.05	0.07	0.06
$k = 3, n = 1$	0.03	0.05	0.05
$k = 3, n = 2$	0.08	0.08	0.07
$k = 3, n = 3$	0.05	0.06	0.05
$k = 4, n = 1$	0.03	0.05	0.05
$k = 4, n = 2$	0.08	0.08	0.07
$k = 4, n = 3$	0.05	0.06	0.05
$k = 5, n = 1$	0.03	0.04	0.04
$k = 5, n = 2$	0.06	0.06	0.06
$k = 5, n = 3$	0.05	0.04	0.04

Table B6. Rest time given to each worker in work shift j.

Bt_j	**Value**
$j = 1$	45
$j = 2$	35
$j = 3$	40

Table B7. Constant parameters.

Parameter	**Value**
(we_1, we_2)	(0.6, 0.4)
$B_{0,n}$	0
λ	0.1
μ_t	0.01
δ_j	8 hrs
O_t	1 hr
U_t	7 hrs

Table B8. Fatigue reduction rate in work shift *j*.

b_j	**Value**
$j = 1$	33
$j = 2$	30
$j = 3$	40

Table B9. Demand of product *n* in period *t*.

$\widetilde{D}_{tn}$	$n = 1$	$n = 2$	$n = 3$
$t = 1$	(3600, 4000, 4500)	(3600, 4000, 4500)	(3600, 4000, 4500)
$t = 2$	(3600, 4000, 4500)	(3600, 4000, 4500)	(3600, 4000, 4500)
$t = 3$	(3600, 4000, 4500)	(3600, 4000, 4500)	(3600, 4000, 4500)
$t = 4$	(3600, 4000, 4500)	(3600, 4000, 4500)	(3600, 4000, 4500)

Table B10. Production cost of each unit of product *n* by contractors in period *t*.

C_{tn}^{sc}	$n = 1$	$n = 2$	$n = 3$
$t = 1$	15	18	19
$t = 2$	15	18	19
$t = 3$	15	18	19
$t = 4$	15	18	19

Table B11. Shortage cost of each unit of product *n* from period *t* to *t* + 1.

C^s_{tn}	$n = 1$	$n = 2$	$n = 3$
$t = 1$	35	40	55
$t = 2$	35	40	55
$t = 3$	35	40	55
$t = 4$	35	40	55

Table B12. Cost per man-hour for working overtime in period *t* for a worker from workforce group *k*.

C^o_{tk}	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$
$t = 1$	22.88	26.12	29.63	42.80	53.15
$t = 2$	22.88	26.12	29.63	42.80	53.15
$t = 3$	22.88	26.12	29.63	42.80	53.15
$t = 4$	22.88	26.12	29.63	42.80	53.15

Table B13. Unemployment cost per man-hour in period *t* for a worker from workforce group *k*.

C^{fire}_{tk}	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$
$t = 1$	35	39.63	45.24	61.21	85.16
$t = 2$	35	39.63	45.24	61.21	85.16
$t = 3$	35	39.63	45.24	61.21	85.16
$t = 4$	35	39.63	45.24	61.21	85.16

Table B14. Maximum production amount of product *n* allowed to be outsourced in period *t*.

SC_{tn}	$n = 1$	$n = 2$	$n = 3$
$t = 1$	500	700	1000
$t = 2$	500	700	1000
$t = 3$	500	700	1000
$t = 4$	500	700	1000

Table B15. Training cost of worker *k* for working in department *n*.

TRC_{kn}	$n = 1$	$n = 2$	$n = 3$
$k = 1$	12.25	12.35	12.55
$k = 2$	14.09	14.20	14.43
$k = 3$	17.12	17.26	17.54
$k = 4$	22.35	22.53	22.89
$k = 5$	31.25	31.51	32.02

Table B16. Internal production cost of each unit of product n (without workforce) in period t.

C_{tn}^{p}	$n = 1$	$n = 2$	$n = 3$
$t = 1$	32	35	40
$t = 2$	32	35	40
$t = 3$	32	35	40
$t = 4$	32	35	40

Table B17. Cost per man-hour for working normally in period t for a worker from workforce group k.

C_{tk}^{r}	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$
$t = 1$	14.28	16.03	18.19	24.06	33.97
$t = 2$	14.28	16.03	18.19	24.06	33.97
$t = 3$	14.28	16.03	18.19	24.06	33.97
$t = 4$	14.28	16.03	18.19	24.06	33.97

Table B18. Cost per man-hour of employment in period t for a worker from workforce group k.

C_{tk}^{hire}	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$
$t = 1$	36	40.32	45.73	55.43	73.23
$t = 2$	36	40.32	45.73	55.43	73.23
$t = 3$	36	40.32	45.73	55.43	73.23
$t = 4$	36	40.32	45.73	55.43	73.23

Table B19. Holding cost of each unit of product n from the period t to $t + 1$.

C_{tn}^{h}	$n = 1$	$n = 2$	$n = 3$
$t = 1$	5	5	7
$t = 2$	5	5	7
$t = 3$	5	5	7
$t = 4$	5	5	7

Table B20. Maximum number of workers in workforce group k.

WU_k	**Value**
$k = 1$	32
$k = 2$	28
$k = 3$	19
$k = 4$	8

$k = 5$	5

Table B21. Minimum number of workers in workforce group k.

WL_k	**Value**
$k = 1$	15
$k = 2$	12
$k = 3$	9
$k = 4$	5
$k = 5$	2

Table B22. Initial number of workers in workforce group k.

$W0_k$	**Value**
$k = 1$	20
$k = 2$	18
$k = 3$	15
$k = 4$	7
$k = 5$	4

Table B23. Required man-hour of workforce group k to produce each unit of product n during normal and overtime working.

m_{njk}	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$
$n = 1, j = 1$	0.040	0.024	0.015	0.010	0.009
$n = 1, j = 2$	0.050	0.030	0.019	0.013	0.011
$n = 1, j = 3$	0.069	0.041	0.026	0.017	0.015
$n = 2, j = 1$	0.066	0.062	0.045	0.020	0.012
$n = 2, j = 2$	0.083	0.078	0.057	0.025	0.016
$n = 2, j = 3$	0.113	0.107	0.077	0.034	0.021
$n = 3, j = 1$	0.049	0.044	0.030	0.016	0.012
$n = 3, j = 2$	0.062	0.056	0.038	0.020	0.015
$n = 3, j = 3$	0.084	0.076	0.051	0.027	0.021

Appendix C. Gams Codes

```
Sets
t/1*4/
n/1*3/
j/1*3/
k/1*5/
g/1*2/
;

alias(t,tp);

Parameters WL(k),WU(k),W0(k) ,U(t), LI(k,n), KI(k,n,j), LE(k,n), LF(k,n,t,j),
Bt(j),  lambda, miu(t), delta(j), D1(t,n),D2(t,n),D3(t,n), CSC(t,n),
Co(t,k),CS(t,n), Cfire(t), O(t), WL, m(n,j,k), W0, Cp(t,n), Cr(t), Ch(t,n),
Chire(t), SC(t,n),WU, I0(n), TRC(k,n), B0(n),MM, bj(j), alpha, gama, we1,we2;

LI(k,n)= uniform(0,6);
KI(k,n,j)=   uniform(8,26);
LE(k,n)=  uniform(0.3,0.7);
LF(k,n,t,j)=   uniform(0.05,0.1);
Bt(j)=  uniform(30,45);
lambda= 0.1;
miu(t)= 0.01;
delta(j)=   480;
D1(t,n)=  3500;
D2(t,n)=  4000;
D3(t,n)= 4500;
alpha=  0.8;
gama=0.3;
CSC(t,n)= uniform(15,19);
Co(t,k)=  uniform(22.88,53.15);
CS(t,n)= uniform(35,55);
Cfire(t)= uniform(35,85.16);
O(t)= 60;
bj(j)= uniform(30,40);
WL(k)=  uniformint(2,15);
W0(k)= uniformint(4,24);
Cp(t,n)= uniform(32,40);
Cr(t,k)=  uniform(14.28, 33.97);
Ch(t,n)=  uniform(5, 7);
Chire(t)=  uniform(36, 73.23);
U(t)=  uniform(480,560);
SC(t,n)= uniform(500,1000);
WU(k)= uniformint(5,32);
I0(n)=  uniform(150,750);
B0(n)=    0 ;
MM=100000000000000 ;
we1= 0.5  ;
```

```
we2=    0.5    ;
Table m(n,j,k)
      1      2      3      4      5
1.1   0.04   0.024  0.015  0.01   0.009
1.2   0.05   0.03   0.019  0.013  0.011
1.3   0.069  0.041  0.026  0.017  0.015
2.1   0.066  0.062  0.045  0.02   0.012
2.2   0.083  0.078  0.057  0.025  0.016
2.3   0.113  0.107  0.077  0.034  0.021
3.1   0.049  0.044  0.03   0.016  0.012
3.2   0.062  0.056  0.038  0.02   0.015
3.3   0.084  0.076  0.051  0.027  0.021
;

Table TRC(k,n)

      1      2      3
1     12.25  12.35  12.55
2     14.09  14.2   14.43
3     17.12  17.26  17.54
4     22.35  22.53  22.89
5     31.25  31.51  32.02

;


Table RR(t,j,k,n)

        1.1       1.2     1.3     2.1     2.2     2.3     3.1     3.2     3.3
4.1     4.2     4.3     5.1    5.2     5.3

1.1     1.1       1.1     1.2     1.6     1.7     1.7     1.6     1.7     1.7
2.2     2.3     2.3     2.2    2.4     2.5
1.2     1         1       1       1.4     1.4     1.5     1.6     1.7     1.8
1.8     1.9     2       1.8    1.9     2.1
1.3     0.9       0.9     0.9     1.2     1.3     1.4     1.4     1.4     1.4
1.4     1.4     1.5     1.4    1.4     1.5
2.1     1.4       1.5     1.5     2       2.2     2.3     2.3     2.3     2.5
2.2     2.5     2.6     2.3    2.5     2.8
2.2     1.3       1.4     1.4     1.7     1.8     1.8     1.9     2       2
1.9     2.1     2.1     1.9    2.1     2.2
2.3     1         1       1.1     1.2     1.2     1.3     1.3     1.4     1.4
1.5     1.5     1.7     1.5    1.6     1.8
3.1     1.3       1.3     1.4     1.7     1.8     2       2       2.1     2.2
2.1     2.2     2.2     2.1    2.2     2.2
3.2     1.1       1.2     1.2     1.3     1.5     1.5     1.5     1.6     1.8
1.7     1.8     1.9     1.7    1.9     1.9
```

```
3.3     0.8      0.9     0.9     1.3     1.3     1.4     1.4     1.4     1.5
1.5     1.5     1.6     1.5    1.5      1.6
4.1     1.1      1.2     1.2     1.6     1.7     1.7     1.8     1.8     1.9
1.8     1.9     2       1.9    2        2.1
4.2     1        1.1     1.1     1.2     1.3     1.3     1.4     1.4     1.5
1.5     1.6     1.6     1.5    1.6      1.7
4.3     0.8      0.8     0.9     1.1     1.2     1.2     1.2     1.2     1.3
1.2     1.3     1.3     1.3    1.3      1.3

;

parameters  b1,b2;
b1=  10202570
;
b2= 171.859
;

variables
z1
z2
WGP
x(t,n), QQSS, yPR(t,n,j), ySC(t,n), I(t,n), B(t,n), MR(t,n,j,k), NR(t,n,j,k),
H1(t,k), H2(t,k), F1(t,k), F2(t,k), WD(t,n), WB(t,n), Wp(t,n),WI(t,n),
XX(t,j,k,n),QQ(t,j,k,n), YY(k,n),d1p,d1n,d2p,d2n,QQreal(t,j,k,n);

positive variables
x(t,n), yPR(t,n,j), ySC(t,n), I(t,n), B(t,n), MR(t,n,j,k), NR(t,n,j,k),
H1(t,k),F1(t,k),  WD(t,n), WB(t,n), Wp(t,n),WI(t,n),QQ(t,j,k,n)
FG(j,t), RG(j,t) , d1p,d1n,d2p,d2n;

integer variables
F2(t,k),H2(t,k);
f2.up(t,k)=10000000000000;
h2.up(t,k)=10000000000000;
binary variables
YY(k,n)
XX(t,j,k,n);

equations
e1
e2
e3
e31 11
e8  16
e9  17
e10 18
e11 19
e12 20
```

```
e13 21
e14 22
e15 23
e16 24
e17 25
e18 26

e20 29
e21 30
e22 31
e24 32
e25 33
e26 34
e27
e28
e311 11

e32 12
e33 12
e34 13
e35 14
e36 15
e37
e38
e39
e4000
e401;


e1.. z1=e=sum((n,t,j),
CP(t,n)*yPR(t,n,j))+sum((t,n),CSC(t,n)*ySC(t,n))+sum((n,t,j,k)$(ord(j)<=2),
cr(t)*MR(t,n,j,k))+sum((n,t,k),CO(t,k)*MR(t,n,'3',k))+
sum((t,n), Ch(t,n)*I(t,n))+sum((t,n), CS(t,n)*B(t,n))+sum((t,k),
Chire(t)*H1(t,k))+sum((t,k), cfire(t)*F1(t,k))+sum((n,k,t), TRC(k,n)*H2(t,k));

e2.. z2=e= ((1-gama)/2)*(sum((t,n,j,k),abs( (((1-FG(j,t))*[ QQreal(t,j,k,n)
])-D1(t,n))/D1(t,n)))+
sum((t,n,j,k),abs( (((1-FG(j,t))* QQreal(t,j,k,n)  )-D2(t,n))/D2(t,n))))
+((gama)/2)*(sum((t,n,j,k),abs( (((1-FG(j,t))*[ QQreal(t,j,k,n)   ])-
D2(t,n))/D2(t,n)))+
sum((t,n,j,k),abs( (((1-FG(j,t))*[ QQreal(t,j,k,n)   ])-D3(t,n))/D3(t,n))));

e3.. WGP=e=we1*(d1p/b1)+we2*(d2p/b2);

e27.. z1-d1p+d1n=e=b1;
e28.. z2-d2p+d2n=e=b2;
```

```
e31(t,n)$(ord(t)=1).. I0(n)+sum(j,yPR(t,n,j))+ySC(t,n)-x(t,n)-B0(n)=e=I(t,n)-
B(t,n);
e311(t,n)$(ord(t)>1).. I(t-1,n)+sum(j,yPR(t,n,j))+ySC(t,n)-x(t,n)-B(t-
1,n)=e=I(t,n)-B(t,n);

e8(t,n,j,k)$(ord(j)<=2).. MR(t,n,j,k)=l=U(t)*NR(t,n,j,k);
e9(t,n,j,k)$(ord(j)=3).. MR(t,n,j,k)=l=O(t)*NR(t,n,j,k);
e10(t,n).. ySC(t,n)=l=SC(t,n);
e11(t,k).. H1(t,k)=g=H2(t,k)*(O(t)+U(t));
e12(t,k).. F1(t,k)=g=F2(t,k)* (O(t)+U(t));


e13(n).. I0(n)-B0(n)+sum(t,x(t,n))=e=sum(t,(1-
alpha)*((D2(t,n)+D3(t,n))/2)+alpha*((D1(t,n)+D2(t,n))/2)          );
e14(t,n).. x(t,n)=e=WD(t,n)+WB(t,n);
e15(t,n).. x(t,n)=e=WP(t,n)+WI(t,n);
e16(t,n)$(ord(t)>1).. WI(t,n)=l=I(t-1,n);
e17(t,n)$(ord(t)>1).. WB(t,n)=l=B(t-1,n);
e18(t,n).. Wp(t,n)=l=sum(j,yPR(t,n,j))+ySC(t,n);

e20(t).. FG('1',t)=e= 1-exp(-lambda*delta('1'));
e21(j,t)$(ord(j)<=2).. FG(j+1,t)=e=RG(j,t)+(1-RG(j,t))*(1-exp(-
lambda*delta(j+1)));
e22(j,t).. RG(j,t)=e=FG(j,t)*exp(-miu(t)*bj(j));
e24(t,j,k,n).. QQ(t,j,k,n)=e=NR(t,n,j,k)* (LI(k,n)+(KI(k,n,j)*(1-
exp(xx(t,j,k,n)/LE(k,n)))*exp(-xx(t,j,k,n)/LF(k,n,t,j))));
*e24(t,j,k,n).. QQ(t,j,k,n)=e=NR(t,n,j,k)*( (LI(k,n))+(KI(k,n,j))*(1-
exp(xx(t,j,k,n)/LE(k,n))**RR(t,j,k,n))*exp(-
xx(t,j,k,n)/LF(k,n,t,j))**RR(t,j,k,n));
*e24(t,j,k,n).. QQ(t,j,k,n)=e=NR(t,n,j,k)*( (LI(k,n))+(KI(k,n,j))*(1-
exp(xx(t,j,k,n)/LE(k,n)))*exp(-xx(t,j,k,n)/LF(k,n,t,j)));


e25(t,n,j).. yPR(t,n,j)=e=sum(k,[ QQreal(t,j,k,n)   ]);
e26(k,n,j,t).. NR(t,n,j,k)=l=MM*xx(t,j,k,n);

e32(k)..W0(k)+H2('1',k)-F2('1',k)=e=sum((j,n),NR('1',n,j,k));
e33(k,t)$(ord(t)>1)..sum((j,n),NR(t-1,n,j,k))+H2(t,k)-
F2(t,k)=e=sum((j,n),NR(t,n,j,k));
e34(t,j,k,n)..yPR(t,n,j)*m(n,j,k)=l=MR(t,n,j,k);
e35(k,t)..sum((j,n),NR(t,n,j,k))=l=WU(k);
e36(k,t)..sum((j,n),NR(t,n,j,k))=g=WL(k);
e37..sum(n,B('4',n))=e=0;
e39..sum(n,I('4',n))=e=0;
e38(t,n)..x(t,n)=l=(1-
alpha)*((D2(t,n)+D3(t,n))/2)+alpha*((D1(t,n)+D2(t,n))/2);
e4000.. QQSS=e=sum((t,n,j,k),[ QQreal(t,j,k,n)   ]);
```

```
e401(t,j,k,n).. QQreal(t,j,k,n)=e=QQ(t,j,k,n)*1.5*RR(t,j,k,n)  ;

model asa
/
all
/;


solve asa using minlp minimizing wgp;

display
WGP.l
z1.l
z2.l
x.l

ySC.l
I.l
B.l

NR.l
H1.l
H2.l
F1.l
F2.l
WD.l
WB.l
Wp.l
WI.l
XX.l
QQreal.l
QQSS.l
d1p.l
d1n.l
d2p.l
d2n.l;
```

Appendix D. MARS Models

Below, some tables and matrices were actually very broad and have been moved inwards and compactified (so that they become tensors), which the expert will basically understand. A reader who is not yet a MARS expert might already be able to get an idea of this extensive series of results.

MARS model for Y_1 in *MQ1*

```
The KEEP list has 7 variables.
```

```
Salford Predictive Modeler(R) software suite: MARS(R) version 8.3.2.001

Data in cache:
N variables: 16
N learn records: 45

The set of model variables appears to have changed.
Checking if they are a subset of the cached data with
consistent coding (continuous, categorical).

The current set of model variables is found
to be a subset of those in the data cache.

                        N
-------------------------
         Learn         45
          Test          0
       Holdout          0
-------------------------
         Total         45

============
MARS Results
============

=================
Distribution of Y
=================

-------------------------
N                      45
Sum(Weights)        45.00
Mean              4.57778
Median            4.00000
Range             8.00000
Sum             206.00000
Cond. Mean        4.57778
Std Dev           2.59798
N = 0                   0
N != 0                 45
-------------------------
MSE               6.59951
RMSE              2.56895
MAD               2.17778
MAPE              0.73058
SSY             296.97778
SSE             296.97778
-------------------------
     Minimum      1.00000
          1%      1.00000
          2%      1.00000
        2.5%      1.00000
          3%      1.00000
          4%      1.00000
          5%      1.00000
         10%      1.00000
         20%      2.00000
         25% Q1   2.00000
         30%      3.00000
         40%      3.50000
-------------------------
      50% Median  4.00000
-------------------------
         60%      5.00000
         70%      7.00000
         75% Q3   7.00000
         80%      7.50000
```

90%	8.00000
95%	9.00000
96%	9.00000
97%	9.00000
97.5%	9.00000
98%	9.00000
99%	9.00000
Maximum	9.00000

================================
Forward Stepwise Knot Placement
================================

BasFn(s)	GCV	IndBsFns	EfPrms	Variable	Knot	Parent	BsF
0	6.90289	0.0	1.0				2
1	7.06315	2.0	5.0	X3	2.70000		4
3	6.65985	4.0	9.0	X4	4.00000		6
5	6.75592	6.0	13.0	X10	7.20000		8
7	7.27126	8.0	17.0	X2	8.60000		10
9	7.77819	10.0	21.0	X13	8.00000		11
	8.66566	11.0	24.0	X11	1.00000		13
12	10.28134	13.0	28.0	X6	6.60000		15
14	13.52472	14.0	31.0	X10	4.00000		

==
Final Model (After Backward Stepwise Elimination)
==

Basis Fun	Coefficient	Variable	Knot	Parent
0	-0.49588			
1	-0.64707	X3	2.70000	
2	-3.37926	X3	2.70000	
4	1.47580	X4	4.00000	
6	1.38077	X10	7.20000	
7	-3.69415	X2	8.60000	
9	-2.12533	X13	8.00000	
11	0.46680	X11	1.00000	
12	-0.76161	X6	6.60000	
14	1.73620	X10	4.00000	

Piecewise Linear GCV = 4.90718, #efprms = 20.28572

===
ANOVA Decomposition on 9 Basis Functions
===

fun	std. dev.	-gcv	#bsfns	#efprms	variable
1	1.64544	8.75133	2	4.28572	X3
2	1.55185	9.97726	1	2.14286	X4
3	1.69991	8.30990	2	4.28572	X10
4	0.80743	5.69898	1	2.14286	X2
5	1.16979	6.67062	1	2.14286	X13
6	1.01221	6.06371	1	2.14286	X11
7	0.78924	5.27943	1	2.14286	X6

===================
Variable Importance
===================

Variable	Importance	-gcv
X4	100.00000	9.97726
X3	87.07487	8.75133
X10	81.92301	8.30990

```
X13                                      58.97576      6.67062
X11                                      47.76069      6.06371
X2                                       39.51856      5.69898
X6                                       27.09637      5.27943

==============================
MARS Regression: Training Data
==============================

W: 45.00                                  R-SQUARED: 0.77572
MEAN DEP VAR: 4.57778                 ADJ R-SQUARED: 0.71805
                  UNCENTERED R-SQUARED = R-0 SQUARED: 0.94629

   Parameter                 Estimate        S.E.      T-Value      P-Value
-----------------------------------------------------------------------------
Constant                 |   -0.49588      1.00939     -0.49127      0.62630
Basis Function 1         |   -0.64707      0.12290     -5.26526      0.00001
Basis Function 2         |   -3.37926      0.50036     -6.75364      0.00000
Basis Function 4         |    1.47580      0.21075      7.00264      0.00000
Basis Function 6         |    1.38077      0.25619      5.38961      0.00000
Basis Function 7         |   -3.69416      1.02451     -3.60579      0.00096
Basis Function 9         |   -2.12533      0.46175     -4.60280      0.00005
Basis Function 11        |    0.46680      0.11643      4.00921      0.00030
Basis Function 12        |   -0.76161      0.24752     -3.07700      0.00405
Basis Function 14        |    1.73620      0.25762      6.73941      0.00000
-----------------------------------------------------------------------------
F-STATISTIC =  13.45051                  S.E. OF REGRESSION =  1.37951
    P-VALUE =  0.00000               RESIDUAL SUM OF SQUARES =  66.60631
  [MDF,NDF] = [ 9, 35 ]            REGRESSION SUM OF SQUARES =  230.37147
-----------------------------------------------------------------------------

===============
Basis Functions
===============

BF1 = max( 0, X3 - 2.7);
BF2 = max( 0, 2.7 - X3);
BF4 = max( 0, 4 - X4);
BF6 = max( 0, 7.2 - X10);
BF7 = max( 0, X2 - 8.6);
BF9 = max( 0, X13 - 8);
BF11 = max( 0, X11 - 1);
BF12 = max( 0, X6 - 6.6);
BF14 = max( 0, X10 - 4);

Y = -0.495878 - 0.647074 * BF1 - 3.37926 * BF2 + 1.4758 * BF4
              + 1.38077 * BF6 - 3.69415 * BF7 - 2.12533 * BF9
              + 0.466798 * BF11 - 0.761607 * BF12 + 1.7362 * BF14;

MODEL Y = BF1 BF2 BF4 BF6 BF7 BF9 BF11 BF12 BF14;

=============
Selector Info
=============

DOF Penalty = 3

BasFn  TotVar DirVar        EffPar          GCV    Learn MSE        Adj MSE    RMSE
----------------------------------------------------------------------------------
    14      7      7      31.00001     13.52473      1.30906        0.87271  1.14414
    13      7      7      28.85715     10.19234      1.31162        0.90356  1.14526
    12      7      7      26.71429      8.15377      1.34635        0.95740  1.16032
    11      7      7      24.57143      6.68952      1.37862        1.01099  1.17415
    10      7      7      22.42857      5.64672      1.42066        1.07339  1.19191
 **  9      7      7      20.28572      4.90718      1.48014        1.15122  1.21661
     8      6      6      18.14286      5.27943      1.88054        1.50443  1.37133
     7      5      5      16.00000      5.60242      2.32673        1.91309  1.52536
     6      4      4      13.85714      5.85187      2.80276        2.36678  1.67415
```

5	3	3	11.71428	6.13136	3.35465	2.90736	1.83157
4	3	3	9.57143	6.20788	3.84791	3.42037	1.96161
3	3	3	7.42857	6.19608	4.31924	3.93531	2.07828
2	2	2	5.28571	5.94257	4.62853	4.31996	2.15140
1	1	1	3.14285	6.35898	5.50176	5.25724	2.34558
0	0	0	1.00000	6.90289	6.59951	.	2.56895

================================
Regression Performance Summary
================================

Sample	Joint N	Wgt Joint N	Mean(Score)	Mean(Target)	R-Sq	RMSE	MSE	MAD	MAPE	Norm R-Sq	SSY	SSE
Lrn	45	45.00	4.57778	4.57778	0.77572	1.21661	1.48014	1.00000	0.34238	0.77572	296.97778	66.60631

==
Performance By Abs(Deviation) Outlier Trimming
==

Percentile	Joint N	Wgt Joint N	Mean(Score)	Mean(Target)	R-Sq	RMSE	MSE	MAD	MAPE	Norm R-Sq	SSY	SSE
Lrn 100%	45	45.00	4.57778	4.57778	0.77572	1.21661	1.48014	1.00000	0.34238	0.77572	296.97778	6.60631
99%	45	45.00	4.57778	4.57778	0.77572	1.21661	1.48014	1.00000	0.34238	0.77572	296.97778	6.60631
98%	45	45.00	4.57778	4.57778	0.77572	1.21661	1.48014	1.00000	0.34238	0.77572	296.97778	6.60631
97.5%	44	44.00	4.54862	4.61364	0.80158	1.15229	1.32778	0.95771	0.32849	0.80242	294.43182	8.42217
97%	44	44.00	4.54862	4.61364	0.80158	1.15229	1.32778	0.95771	0.32849	0.80242	294.43182	8.42217
96%	44	44.00	4.54862	4.61364	0.80158	1.15229	1.32778	0.95771	0.32849	0.80242	294.43182	8.42217
95%	43	43.00	4.49864	4.62791	0.82606	1.09061	1.18943	0.91725	0.32044	0.82997	294.04651	1.14548
90%	41	41.00	4.45162	4.48780	0.84267	1.02208	1.04466	0.86261	0.32216	0.84353	272.24390	2.83088
80%	36	36.00	4.59693	4.69444	0.88640	0.89465	0.80040	0.75021	0.27978	0.88977	253.63889	8.81430
75% Q3	34	34.00	4.45699	4.47059	0.89139	0.84301	0.71066	0.70468	0.28568	0.89163	222.47059	4.16255
70%	32	32.00	4.45422	4.46875	0.90287	0.79833	0.63733	0.66294	0.27594	0.90290	209.96875	0.39443
60%	27	27.00	4.70077	4.66667	0.93243	0.65609	0.43045	0.54045	0.21015	0.93546	172.00000	11.62226
50% Median	23	23.00	4.81135	4.78261	0.95930	0.51846	0.26880	0.43213	0.16282	0.96166	151.91304	6.18233
40%	18	18.00	5.15595	5.11111	0.97811	0.37194	0.13834	0.31721	0.09663	0.98341	113.77778	2.49006
30%	14	14.00	5.55228	5.57143	0.98309	0.29369	0.08626	0.24620	0.04564	0.98371	71.42857	1.20758
25% Q1	12	12.00	5.22634	5.16667	0.98703	0.24526	0.06015	0.20521	0.04290	0.98792	55.66667	0.72184
20%	9	9.00	4.71413	4.66667	0.99447	0.16814	0.02827	0.14273	0.03758	0.99493	46.00000	0.25443
10%	5	5.00	4.01881	4.00000	0.99883	0.08368	0.00700	0.07440	0.03364	0.99895	30.00000	0.03501
5%	3	3.00	4.62315	4.66667	0.99960	0.05702	0.00325	0.04914	0.02766	0.99984	24.66667	0.00975
4%	2	2.00	6.47126	6.50000	0.99902	0.04699	0.00221	0.03718	0.00496	1.00000	4.50000	0.00442
3%	2	2.00	6.47126	6.50000	0.99902	0.04699	0.00221	0.03718	0.00496	1.00000	4.50000	0.00442

2.5%	2	2.00	6.47126	6.50000	0.99902	0.04699	0.00221	0.03718	0.00496	1.00000	4.50000	0.00442
2%	1	1.00	5.00844	5.00000	.	0.00844	0.00007	0.00844	0.00169	.	0.00000	0.00007
1%	1	1.00	5.00844	5.00000	.	0.00844	0.00007	0.00844	0.00169	.	0.00000	0.00007

97.78%	-1	44.00	4.54862	4.61364	0.80158	1.15229	1.32778	0.95771	0.32849	0.80242	294.43182	8.42217
88.89%	-5	40.00	4.44230	4.52500	0.85368	0.99375	0.98755	0.83856	0.31501	0.85552	269.97500	9.50181
77.78%	-10	35.00	4.51614	4.57143	0.88775	0.86737	0.75233	0.72663	0.28277	0.88897	234.57143	6.33144
44.44%	-25	20.00	5.12192	5.15000	0.97292	0.41397	0.17137	0.35392	0.10285	0.97642	126.55000	3.42748

Percentage of Error Statistics Due To Outliers

	% Outliers	% MAD	Lift(MAD)	% MSE	Lift(MSE)	% MAPE	N	Wgt N
Lrn	1%	6.36	6.36	12.29	12.29	8.63	1	1.00
	2%	6.36	3.18	12.29	6.14	8.63	1	1.00
	2.5%	12.35	4.94	23.21	9.28	16.33	2	2.00
	3%	12.35	4.12	23.21	7.74	16.33	2	2.00
	4%	12.35	3.09	23.21	5.80	16.33	2	2.00
	5%	17.05	3.41	29.94	5.99	22.98	3	3.00
	10%	25.46	2.55	40.69	4.07	34.72	5	5.00
	20%	39.98	2.00	56.74	2.84	53.35	9	9.00
	25% Q1	49.81	1.99	66.56	2.66	65.26	12	12.00
	30%	55.88	1.86	72.16	2.41	71.34	14	14.00
	40%	67.57	1.69	82.55	2.06	79.87	18	18.00
	50% Median	80.19	1.60	92.29	1.85	87.86	23	23.00
	60%	87.31	1.46	96.26	1.60	92.69	27	27.00
	70%	93.49	1.34	98.59	1.41	96.33	32	32.00
	75% Q3	95.52	1.27	99.21	1.32	97.17	34	34.00
	80%	97.15	1.21	99.62	1.25	97.92	36	36.00
	90%	99.43	1.10	99.97	1.11	99.54	41	41.00
	95%	99.83	1.05	99.99	1.05	99.94	43	43.00
	96%	99.98	1.04	100.00	1.04	99.99	44	44.00
	97%	99.98	1.03	100.00	1.03	99.99	44	44.00
	97.5%	99.98	1.03	100.00	1.03	99.99	44	44.00
	98%	100.00	1.02	100.00	1.02	100.00	45	45.00

```
       99%       100.00         1.01       100.00         1.01       100.00
45          45.00
      100%       100.00         1.00       100.00         1.00       100.00
45          45.00
--------------------------------------------------------------------------------
     2.22%         6.36         2.86        12.29         5.53         8.63
1           1.00
    11.11%        25.46         2.29        40.69         3.66        34.72
5           5.00
    22.22%        43.48         1.96        60.47         2.72        57.60
10          10.00
    55.56%        84.27         1.52        94.85         1.71        90.36
25          25.00
--------------------------------------------------------------------------------

 =================================================
 Learn Sample Residual Fit Diagnostics - 9-BF Model
 =================================================

             Mean          Min          Max        Wgt N
 -------------------------------------------------------
 Y        4.57778      1.00000      9.00000        45.00
 YHat     4.57778     -0.32974      8.48111        45.00

 --------- Predicted Response -------          ------ Standardized Residual -------

      N            W       Mean(Y)         Mean          Min          Max
StdDev          IQ1          IQ3
 --------------------------------------------------------------------------------
      3         3.00      1.33333      0.41197     -0.32974      0.92695
0.49316      0.06005      1.11893
      3         3.00      2.00000      1.39058      1.18102      1.57219
0.73288     -0.47032      1.29990
      3         3.00      1.33333      2.10883      2.02390      2.18633
0.38701     -0.97511     -0.09556
      3         3.00      3.00000      2.59538      2.29564      2.96067
0.55462     -0.43550      0.85429
      3         3.00      3.00000      3.36597      3.30853      3.41539
1.13391     -1.12936      1.30248
      2         2.00      2.50000      3.61152      3.56708      3.65597
0.44751     -1.36113     -0.46612
      2         2.00      4.00000      3.79815      3.71268      3.88362
1.57366     -1.40774      1.73957
      2         2.00      4.00000      4.14027      4.10835      4.17218
0.02623     -0.14153     -0.08906
      2         2.00      3.50000      4.50179      4.17901      4.82457
0.67629     -1.49972     -0.14714
      2         2.00      4.50000      5.01308      5.00844      5.01772
0.41479     -0.83652     -0.00694
      2         2.00      4.00000      5.28982      5.27008      5.30955
0.01622     -1.07639     -1.04395
      2         2.00      6.50000      5.66952      5.62876      5.71028
0.44448      0.23813      1.12710
      2         2.00      5.00000      5.85177      5.84275      5.86079
1.65133     -2.35145      0.95121
      2         2.00      7.00000      6.43681      6.34634      6.52729
0.74759     -0.28467      1.21051
      2         2.00      7.00000      6.54508      6.53471      6.55545
0.00852      0.36540      0.38244
      2         2.00      6.00000      6.69233      6.68713      6.69754
1.64819     -2.21726      1.07913
      2         2.00      8.50000      7.18862      7.04155      7.33569
0.53186      0.54603      1.60976
      2         2.00      7.50000      7.39658      7.38709      7.40606
0.40318     -0.31817      0.48819
      2         2.00      8.50000      7.67918      7.42429      7.93408
0.62049      0.05418      1.29516
```

```
     2        2.00      8.50000      8.37638      8.27165      8.48111
0.32490     -0.22329      0.42651
-------------------------------------------------------------------------------------
    45       45.00

 Grove file created: C:\Users\ayseo\AppData\Local\Temp\vi4_00454.grv: 89 kb,
 75% compression

 Grove file created containing:
      1 Mars model

 Import processed data cache : 00:00:00
 MARS model building         : 00:00:00
 Total                       : 00:00:00
>REM
>
```

MARS model for Y_2 in *MQ1*

```
The KEEP list has 5 variables.

 Salford Predictive Modeler(R) software suite: MARS(R) version 8.3.2.001

 Data in cache:
 N variables: 16
 N learn records: 45

 The set of model variables appears to have changed.
 Checking if they are a subset of the cached data with
 consistent coding (continuous, categorical).

 The current set of model variables is found
 to be a subset of those in the data cache.

                        N
 -------------------------
         Learn         45
          Test          0
       Holdout          0
 -------------------------
         Total         45

 ============
 MARS Results
 ============

 =================
 Distribution of Y
 =================

 -------------------------
 N                     45
 Sum(Weights)       45.00
 Mean             4.24444
 Median           4.00000
 Range            9.00000
 Sum            191.00000
 Cond. Mean       4.24444
 Std Dev          2.55979
 N = 0                  0
 N != 0                45
 -------------------------
 MSE              6.40691
 RMSE             2.53119
 MAD              2.06667
 MAPE             0.74582
```

SSY	288.31111
SSE	288.31111

Minimum	1.00000
1%	1.00000
2%	1.00000
2.5%	1.00000
3%	1.00000
4%	1.00000
5%	1.00000
10%	1.00000
20%	2.00000
25% Q1	2.00000
30%	2.00000
40%	3.00000
50% Median	4.00000
60%	4.50000
70%	5.00000
75% Q3	6.00000
80%	6.50000
90%	8.00000
95%	9.00000
96%	9.00000
97%	9.00000
97.5%	9.00000
98%	10.00000
99%	10.00000
Maximum	10.00000

================================
Forward Stepwise Knot Placement
================================

BasFn(s)	GCV	IndBsFns	EfPrms	Variable	Knot	Parent	BsF
0	6.70145	0.0	1.0				2
1	7.10960	2.0	6.0	X6	3.00000		3
	7.77259	3.0	10.0	X11	1.40000	X6	5
4	9.02905	5.0	15.0	X13	6.00000		7
6	9.00620	7.0	20.0	X11	5.00000	X13	9
8	11.46037	9.0	25.0	X9	2.00000		11
10	13.31254	11.0	30.0	X5	3.00000		13
12	20.85529	13.0	35.0	X11	7.10000	X13	4
14	51.21160	14.0	39.0	X9	1.00000	X5	16
15	1235.51990	16.0	44.0	X11	6.50000		

==
Final Model (After Backward Stepwise Elimination)
==

Basis Fun	Coefficient	Variable	Knot	Parent
0	3.29736			
2	9.25319	X6	3.00000	
3	-2.23677	X11	1.40000	X6
7	-0.48673	X11	5.00000	X13
9	2.99137	X9	2.00000	
11	1.36294	X5	3.00000	
12	-1.55822	X11	7.10000	X13
15	1.68888	X11	6.50000	

Piecewise Linear GCV = 4.28984, #efprms = 19.81250

==
ANOVA Decomposition on 7 Basis Functions
==

```
 fun     std. dev.         -gcv #bsfns  #efprms variable
 -------------------------------------------------------
   1      3.25254     12.20256   1      2.68750 X6
   2      0.86841      5.07196   1      2.68750 X9
   3      0.72320      4.68319   1      2.68750 X5
   4      1.55223      6.85290   1      2.68750 X11
   5      2.54489      9.08979   1      2.68750 X6
                                                X11
   6      2.07859      9.17666   2      5.37500 X11
                                                X13
```

```
===================
 Variable Importance
 ===================

 Variable                                Importance         -gcv
 ---------------------------------------------------------------
 X6                                       100.00000     10.15339
 X13                                       91.29201      9.17666
 X11                                       82.56674      8.28718
 X9                                        36.52206      5.07196
 X5                                        25.90066      4.68319
```

```
==============================
MARS Regression: Training Data
==============================

W: 45.00                                   R-SQUARED: 0.79023
MEAN DEP VAR: 4.24444                  ADJ R-SQUARED: 0.75055
                  UNCENTERED R-SQUARED = R-0 SQUARED: 0.94497

   Parameter                  Estimate        S.E.      T-Value      P-Value
-----------------------------------------------------------------------------
Constant               |       3.29736     0.29383     11.22195      0.00000
Basis Function 2       |       9.25319     0.96521      9.58673      0.00000
Basis Function 3       |      -2.23677     0.29115     -7.68262      0.00000
Basis Function 7       |      -0.48673     0.08362     -5.82088      0.00000
Basis Function 9       |       2.99137     0.73467      4.07175      0.00024
Basis Function 11      |       1.36294     0.38593      3.53162      0.00113
Basis Function 12      |      -1.55822     0.22057     -7.06456      0.00000
Basis Function 15      |       1.68888     0.28389      5.94918      0.00000
-----------------------------------------------------------------------------
F-STATISTIC =  19.91229                  S.E. OF REGRESSION =  1.27849
    P-VALUE =  0.00000               RESIDUAL SUM OF SQUARES =  60.47821
  [MDF,NDF] = [ 7, 37 ]            REGRESSION SUM OF SQUARES =  227.83290
-----------------------------------------------------------------------------
```

```
===============
Basis Functions
===============

BF2 = max( 0, 3 - X6);
BF3 = max( 0, X11 - 1.4) * BF2;
BF4 = max( 0, X13 - 6);
BF5 = max( 0, 6 - X13);
BF7 = max( 0, 5 - X11) * BF5;
BF9 = max( 0, 2 - X9);
BF11 = max( 0, 3 - X5);
BF12 = max( 0, X11 - 7.1) * BF4;
BF15 = max( 0, X11 - 6.5);

Y = 3.29736 + 9.25319 * BF2 - 2.23677 * BF3 - 0.486727 * BF7
            + 2.99137 * BF9 + 1.36294 * BF11 - 1.55822 * BF12
            + 1.68888 * BF15;

MODEL Y = BF2 BF3 BF7 BF9 BF11 BF12 BF15;
```

==============
Selector Info
==============

DOF Penalty = 3

BasFn	TotVar	DirVar	EffPar	GCV	Learn MSE	Adj MSE	RMSE
16	5	5	44.00000	1235.51989	0.61013	0.37964	0.78111
15	5	5	41.31250	90.86417	0.61014	0.39320	0.78112
14	5	5	38.62500	30.61573	0.61444	0.40963	0.78386
13	5	5	35.93750	15.27609	0.61956	0.42681	0.78712
12	5	5	33.25000	9.24323	0.63019	0.44814	0.79385
11	5	5	30.56250	6.32944	0.65152	0.47778	0.80716
10	5	5	27.87500	4.55421	0.65955	0.49833	0.81213
9	5	5	25.18750	4.74399	0.91960	0.71524	0.95896
8	5	5	22.50000	4.41844	1.10461	0.88369	1.05100
** 7	5	5	19.81250	4.28984	1.34396	1.10503	1.15929
6	4	4	17.12500	4.68319	1.79700	1.51746	1.34052
5	3	3	14.43750	5.13027	2.36642	2.05090	1.53832
4	3	3	11.75000	5.20827	2.84349	2.52755	1.68626
3	3	3	9.06250	6.21243	3.96216	3.60997	1.99052
2	3	3	6.37500	6.67612	4.91854	4.59064	2.21778
1	1	1	3.68750	6.40786	5.40071	5.16068	2.32394
0	0	0	1.00000	6.70145	6.40691	.	2.53119

==============================
Regression Performance Summary
==============================

Sample	Joint N	Wgt Joint N	Mean(Score)	Mean(Target)	R-Sq	RMSE	MSE	MAD	MAPE	Norm R-Sq	SSY	SSE
Lrn	45	45.00	4.24444	4.24444	0.79023	1.15929	1.34396	0.91862	0.35131	0.79023	288.31111	60.47821

==
Performance By Abs(Deviation) Outlier Trimming
==

Percentile	Joint N	Wgt Joint N	Mean(Score)	Mean(Target)	R-Sq	RMSE	MSE	MAD	MAPE	Norm R-Sq	SSY	SSE
Lrn 100%	45	45.00	4.24444	4.24444	0.79023	1.15929	1.34396	0.91862	0.35131	0.79023	288.31111	60.47821
99%	45	45.00	4.24444	4.24444	0.79023	1.15929	1.34396	0.91862	0.35131	0.79023	288.31111	60.47821
98%	45	45.00	4.24444	4.24444	0.79023	1.15929	1.34396	0.91862	0.35131	0.79023	288.31111	60.47821
97.5%	44	44.00	4.24294	4.31818	0.82159	1.06085	1.12540	0.86426	0.28405	0.82249	277.54545	49.51751
97%	44	44.00	4.24294	4.31818	0.82159	1.06085	1.12540	0.86426	0.28405	0.82249	277.54545	49.51751
96%	44	44.00	4.24294	4.31818	0.82159	1.06085	1.12540	0.86426	0.28405	0.82249	277.54545	49.51751
95%	43	43.00	4.22319	4.23256	0.84427	0.97721	0.95493	0.81673	0.28220	0.84437	263.67442	41.06213
90%	41	41.00	4.28271	4.26829	0.87667	0.86727	0.75216	0.74898	0.24331	0.87680	250.04878	30.83866
80%	36	36.00	4.28208	4.30556	0.91348	0.73313	0.53749	0.64292	0.20784	0.91451	223.63889	19.34952
75% Q3	34	34.00	4.27988	4.38235	0.92723	0.68312	0.46665	0.60312	0.19112	0.92980	218.02941	15.86599
70%	32	32.00	4.29517	4.40625	0.93733	0.64089	0.41074	0.56793	0.17914	0.93922	209.71875	13.14372
60%	27	27.00	4.11780	4.22222	0.95356	0.53540	0.28665	0.48118	0.16193	0.95544	166.66667	7.73967

```
      50% Median  23        23.00       3.47397      3.52174       0.93258
0.47119       0.22202      0.42387      0.17158      0.93455     75.73913     5.10654
        40%         18        18.00       3.55606      3.50000       0.96252
0.38853       0.15095      0.34963      0.16188      0.96330     72.50000     2.71716
        30%         14        14.00       3.58002      3.50000       0.97205
0.29468       0.08684      0.27469      0.12352      0.97654     43.50000     1.21570
        25% Q1      12        12.00       3.75122      3.58333       0.97664
0.26072       0.06798      0.24594      0.09916      0.98703     34.91667     0.81572
        20%          9         9.00       4.30481      4.11111       0.97978
0.22674       0.05141      0.21410      0.06135      0.99458     22.88889     0.46272
        10%          5         5.00       5.11602      5.00000       0.99115
0.15743       0.02479      0.15274      0.03288      0.99937     14.00000     0.12393
         5%          3         3.00       4.39887      4.33333       0.99420
0.12944       0.01675      0.12675      0.03305      0.99962      8.66667     0.05026
         4%          2         2.00       4.02074      4.00000       0.99673
0.11445       0.01310      0.11255      0.03406      1.00000      8.00000     0.02620
         3%          2         2.00       4.02074      4.00000       0.99673
0.11445       0.01310      0.11255      0.03406      1.00000      8.00000     0.02620
       2.5%          2         2.00       4.02074      4.00000       0.99673
0.11445       0.01310      0.11255      0.03406      1.00000      8.00000     0.02620
         2%          1         1.00       1.90819      2.00000        .
0.09181       0.00843      0.09181      0.04590       .            0.00000     0.00843
         1%          1         1.00       1.90819      2.00000        .
0.09181       0.00843      0.09181      0.04590       .            0.00000     0.00843
-----------------------------------------------------------------------------------------
     97.78%         -1        44.00       4.24294      4.31818       0.82159
1.06085       1.12540      0.86426      0.28405      0.82249    277.54545     9.51751
     88.89%         -5        40.00       4.23128      4.17500       0.88089
0.83790       0.70208      0.72621      0.24420      0.88208    235.77500     8.08311
     77.78%        -10        35.00       4.25181      4.31429       0.92150
0.70810       0.50140      0.62295      0.20420      0.92268    223.54286     7.54915
     44.44%        -25        20.00       3.50046      3.45000       0.95163
0.42576       0.18127      0.38206      0.17096      0.95325     74.95000     3.62545

 ==================================================
 Percentage of Error Statistics Due To Outliers
 ==================================================

 % Outliers          % MAD     Lift(MAD)         % MSE     Lift(MSE)         % MAPE
N         Wgt N
-----------------------------------------------------------------------------------------
 Lrn      1%          8.01          8.01         18.12         18.12          20.94
1          1.00
          2%          8.01          4.00         18.12          9.06          20.94
1          1.00
        2.5%         15.04          6.02         32.10         12.84          31.75
2          2.00
          3%         15.04          5.01         32.10         10.70          31.75
2          2.00
          4%         15.04          3.76         32.10          8.03          31.75
2          2.00
          5%         21.58          4.32         44.18          8.84          36.80
3          3.00
         10%         29.73          2.97         53.56          5.36          45.44
5          5.00
         20%         44.01          2.20         68.01          3.40          59.45
9          9.00
         25% Q1      53.30          2.13         76.15          3.05          67.45
12         12.00
         30%         58.75          1.96         80.35          2.68          71.91
14         14.00
         40%         68.57          1.71         87.20          2.18          79.93
18         18.00
         50% Median  78.12          1.56         92.37          1.85          86.29
23         23.00
         60%         84.78          1.41         95.51          1.59          90.69
27         27.00
```

```
       70%        91.79        1.31        98.33        1.40        94.42
32         32.00
        75% Q3    93.77        1.25        98.89        1.32        95.67
34         34.00
       80%        95.34        1.19        99.23        1.24        96.84
36         36.00
       90%        98.64        1.10        99.86        1.11        99.25
41         41.00
       95%        99.46        1.05        99.96        1.05        99.68
43         43.00
       96%        99.78        1.04        99.99        1.04        99.86
44         44.00
       97%        99.78        1.03        99.99        1.03        99.86
44         44.00
       97.5%      99.78        1.02        99.99        1.03        99.86
44         44.00
       98%       100.00        1.02       100.00        1.02       100.00
45         45.00
       99%       100.00        1.01       100.00        1.01       100.00
45         45.00
      100%       100.00        1.00       100.00        1.00       100.00
45         45.00
-------------------------------------------------------------------------------
     2.22%         8.01        3.60        18.12        8.16        20.94
1         1.00
    11.11%        29.73        2.68        53.56        4.82        45.44
5         5.00
    22.22%        47.26        2.13        70.98        3.19        62.30
10         10.00
    55.56%        81.52        1.47        94.01        1.69        88.52
25         25.00
-------------------------------------------------------------------------------


 ================================================
 Learn Sample Residual Fit Diagnostics - 7-BF Model
 ================================================

             Mean          Min          Max          Wgt N
 ------------------------------------------------------
 Y         4.24444      1.00000     10.00000        45.00
 YHat      4.24444      0.54941     10.76114        45.00

--------- Predicted Response -------          ------ Standardized Residual -------

      N            W       Mean(Y)        Mean          Min           Max
StdDev          IQ1          IQ3
--------------------------------------------------------------------------------
      1         1.00      1.00000      0.54941      0.54941      0.54941
0.00000      0.38868      0.38868
      2         2.00      2.00000      1.09761      0.86910      1.32612
0.19711      0.58128      0.97551
      2         2.00      1.50000      1.36019      1.35046      1.36993
0.42290     -0.30230      0.54350
      2         2.00      1.50000      1.61092      1.59834      1.62350
0.42045     -0.51613      0.32477
      2         2.00      2.50000      1.92191      1.90819      1.93563
0.41946      0.07919      0.91812
      2         2.00      1.50000      2.91536      2.70858      3.12214
0.25293     -1.47381     -0.96795
      1         1.00      3.00000      3.27113      3.27113      3.27113
0.00000     -0.23388     -0.23388
     10        10.00      3.40000      3.31099      3.29736      3.43366
0.97585     -0.25650      0.60609
      1         1.00      3.00000      3.56995      3.56995      3.56995
0.00000     -0.49164     -0.49164
      2         2.00      3.50000      3.59704      3.59650      3.59759
1.29436     -1.37807      1.21065
```

```
       2         2.00       3.00000      4.23932      4.16794      4.31069
1.78676     -2.85579       0.71773
       2         2.00       4.00000      4.51229      4.46837      4.55621
0.82471     -1.26661       0.38281
       2         2.00       6.00000      4.88306      4.67393      5.09219
1.54479     -0.58132       2.50827
       2         2.00       5.00000      5.16915      5.15514      5.18317
0.01209     -0.15800      -0.13382
       2         2.00       4.50000      5.72144      5.34178      6.10111
0.10380     -1.15741      -0.94981
       2         2.00       7.00000      6.23666      6.13330      6.34002
0.77344     -0.11498       1.43189
       2         2.00       7.50000      7.10030      7.00031      7.20029
0.51755     -0.17277       0.86233
       2         2.00       7.00000      7.27604      7.20139      7.35068
0.79821     -1.03632       0.56010
       1         1.00       9.00000      8.10549      8.10549      8.10549
0.00000      0.77160       0.77160
       3         3.00       9.33333      9.03716      8.15523     10.76114
0.64505     -0.65656       0.72870
-----------------------------------------------------------------------------------
      45        45.00

 Grove file created: C:\Users\ayseo\AppData\Local\Temp\vi4_00628.grv: 88 kb,
 79% compression

 Grove file created containing:
      1 Mars model

 Import processed data cache : 00:00:00
 MARS model building         : 00:00:02
 Total                       : 00:00:02
>REM
>
```

MARS model for Y_3 in *MQ1*

```
The KEEP list has 5 variables.

 Salford Predictive Modeler(R) software suite: MARS(R) version 8.3.2.001

 Data in cache:
 N variables: 16
 N learn records: 45

 The set of model variables appears to have changed.
 Checking if they are a subset of the cached data with
 consistent coding (continuous, categorical).

 The current set of model variables is found
 to be a subset of those in the data cache.

                         N
 -------------------------
          Learn         45
           Test          0
        Holdout          0
 -------------------------
          Total         45

 ============
 MARS Results
 ============

 =================
 Distribution of Y
```

```
=================

-------------------------
N                       45
Sum(Weights)         45.00
Mean               4.93333
Median             5.00000
Range              9.00000
Sum              222.00000
Cond. Mean         4.93333
Std Dev            2.26033
N = 0                    0
N != 0                  45
-------------------------
MSE                4.99556
RMSE               2.23507
MAD                1.80000
MAPE               0.61512
SSY              224.80000
SSE              224.80000
-------------------------
     Minimum       1.00000
          1%       1.00000
          2%       1.00000
        2.5%       1.00000
          3%       1.00000
          4%       1.00000
          5%       1.00000
         10%       2.00000
         20%       3.00000
      25% Q1       3.00000
         30%       3.00000
         40%       4.00000
-------------------------
  50% Median       5.00000
-------------------------
         60%       5.00000
         70%       6.00000
      75% Q3       7.00000
         80%       7.00000
         90%       8.00000
         95%       9.00000
         96%       9.00000
         97%       9.00000
       97.5%       9.00000
         98%      10.00000
         99%      10.00000
     Maximum      10.00000

===============================
Forward Stepwise Knot Placement
===============================

BasFn(s)       GCV  IndBsFns  EfPrms  Variable                      Knot   Parent   BsF
-----------------------------------------------------------------------------------
    0        5.22521     0.0     1.0                                                  2
    1        5.27607     2.0     6.0  X11                        2.10000              4
    3        5.42797     4.0    11.0  X1                         2.40000   X11        6
    5        5.73261     6.0    16.0  X5                         2.40000              8
    7        6.74668     8.0    21.0  X1                         2.90000   X5         5
    9        7.55970     9.0    25.0  X15                        2.00000             11
   10       10.31211    11.0    30.0  X8                         5.50000   X15        9
   12       18.32504    12.0    34.0  X1                         1.00000   X15       14
   13       54.33902    14.0    39.0  X15                        4.10000   X5         5
   15      472.15744    15.0    43.0  X8                         1.00000   X5         5

=================================================
Final Model (After Backward Stepwise Elimination)
```

```
=================================================

Basis Fun  Coefficient Variable                   Knot   Parent
-----------------------------------------------------------------
        0      3.49149
        2     -2.37037 X11                        2.10000
        4     -1.56803 X1                         2.40000  X11
        6      5.07056 X5                         2.40000
        8      0.53340 X1                         2.90000  X5
        9      0.91494 X15                        2.00000
       10     -0.11152 X8                         5.50000  X15
       12     -0.15671 X1                         1.00000  X15

Piecewise Linear GCV = 4.50814, #efprms = 20.60001

========================================
ANOVA Decomposition on 7 Basis Functions
========================================

 fun    std. dev.         -gcv #bsfns  #efprms variable
-------------------------------------------------------
   1      0.79247      4.95896   1      2.80000 X11
   2      1.14612      6.47439   1      2.80000 X5
   3      1.87400      7.67836   1      2.80000 X15
   4      1.12265      5.89424   1      2.80000 X1
                                               X11
   5      1.02139      5.61421   1      2.80000 X1
                                               X5
   6      0.73047      4.72789   1      2.80000 X8
                                               X15
   7      1.37336      5.85859   1      2.80000 X1
                                               X15

===================
Variable Importance
===================

Variable                                Importance         -gcv
---------------------------------------------------------------
X1                                       100.00000      6.57480
X5                                        84.31700      5.97740
X15                                       66.93432      5.43404
X11                                       59.61614      5.24265
X8                                        32.60839      4.72789

==============================
MARS Regression: Training Data
==============================

W: 45.00                                   R-SQUARED: 0.73468
MEAN DEP VAR: 4.93333                  ADJ R-SQUARED: 0.68449
                  UNCENTERED R-SQUARED = R-0 SQUARED: 0.95482

   Parameter                  Estimate          S.E.      T-Value       P-Value
-------------------------------------------------------------------------------
Constant                 |     3.49149       0.55900      6.24601       0.00000
Basis Function 2         |    -2.37037       0.64330     -3.68470       0.00073
Basis Function 4         |    -1.56803       0.32614     -4.80791       0.00003
Basis Function 6         |     5.07056       0.94104      5.38823       0.00000
Basis Function 8         |     0.53340       0.11851      4.50111       0.00007
Basis Function 9         |     0.91494       0.14235      6.42748       0.00000
Basis Function 10        |    -0.11152       0.03329     -3.34967       0.00187
Basis Function 12        |    -0.15671       0.03285     -4.76996       0.00003
-------------------------------------------------------------------------------
F-STATISTIC =  14.63641                   S.E. OF REGRESSION =  1.26964
    P-VALUE =  0.00000               RESIDUAL SUM OF SQUARES =  59.64366
  [MDF,NDF] = [ 7, 37 ]            REGRESSION SUM OF SQUARES =  165.15634
-------------------------------------------------------------------------------
```

```
================
Basis Functions
================

BF1 = max( 0, X11 - 2.1);
BF2 = max( 0, 2.1 - X11);
BF4 = max( 0, 2.4 - X1) * BF1;
BF5 = max( 0, X5 - 2.4);
BF6 = max( 0, 2.4 - X5);
BF8 = max( 0, 2.9 - X1) * BF5;
BF9 = max( 0, X15 - 2);
BF10 = max( 0, X8 - 5.5) * BF9;
BF12 = max( 0, X1 - 1) * BF9;

Y = 3.49149 - 2.37037 * BF2 - 1.56803 * BF4 + 5.07056 * BF6
            + 0.533405 * BF8 + 0.914942 * BF9 - 0.111522 * BF10
            - 0.156712 * BF12;

MODEL Y = BF2 BF4 BF6 BF8 BF9 BF10 BF12;

=============
Selector Info
=============

DOF Penalty = 3

BasFn  TotVar DirVar      EffPar          GCV    Learn MSE     Adj MSE   RMSE
-------------------------------------------------------------------------------------
    15      5      5    43.00000    472.15745      0.93266      0.60105  0.96574
    14      5      5    40.20000     81.98229      0.93278      0.62185  0.96580
    13      5      5    37.40000     32.73953      0.93384      0.64331  0.96636
    12      5      5    34.60000     17.68186      0.94443      0.67159  0.97182
    11      5      5    31.80000     11.48057      0.98784      0.72442  0.99390
    10      5      5    29.00000      8.38130      1.05956      0.80056  1.02935
     9      5      5    26.20000      6.83051      1.19218      0.92725  1.09187
     8      5      5    23.40001      5.39639      1.24333      0.99466  1.11505
 **  7      5      5    20.60001      4.50814      1.32541      1.08979  1.15127
     6      4      4    17.80001      4.72789      1.72735      1.45865  1.31429
     5      4      4    15.00001      4.85427      2.15745      1.86979  1.46883
     4      4      4    12.20001      4.87800      2.59158      2.30363  1.60984
     3      3      3     9.40001      4.67291      2.92457      2.66461  1.71014
     2      2      2     6.60001      5.46710      3.98102      3.71562  1.99525
     1      1      1     3.80001      5.47029      4.58543      4.38163  2.14136
     0      0      0     1.00000      5.22521      4.99556         .      2.23507
-------------------------------------------------------------------------------------

==============================
Regression Performance Summary
==============================

Sample       Joint N  Wgt Joint N  Mean(Score) Mean(Target)         R-Sq
RMSE           MSE           MAD         MAPE     Norm R-Sq            SSY        SSE
-------------------------------------------------------------------------------------
Lrn               45        45.00      4.93333      4.93333      0.73468
1.15127      1.32541      0.94624      0.25654      0.73468    224.80000   59.64366

==============================================
Performance By Abs(Deviation) Outlier Trimming
==============================================

Percentile   Joint N  Wgt Joint N  Mean(Score) Mean(Target)         R-Sq
RMSE           MSE           MAD         MAPE     Norm R-Sq           SSY         SSE
-------------------------------------------------------------------------------------
Lrn   100%        45        45.00      4.93333      4.93333      0.73468
1.15127      1.32541      0.94624      0.25654      0.73468    224.80000  59.64366
       99%        45        45.00      4.93333      4.93333      0.73468
1.15127      1.32541      0.94624      0.25654      0.73468    224.80000  59.64366
```

```
         98%        45         45.00       4.93333       4.93333       0.73468
1.15127       1.32541       0.94624       0.25654       0.73468     224.80000   59.64366
       97.5%        44         44.00       4.87229       4.93182       0.76520
1.09527       1.19961       0.90821       0.25047       0.76735     224.79545   52.78285
         97%        44         44.00       4.87229       4.93182       0.76520
1.09527       1.19961       0.90821       0.25047       0.76735     224.79545   52.78285
         96%        44         44.00       4.87229       4.93182       0.76520
1.09527       1.19961       0.90821       0.25047       0.76735     224.79545   52.78285
         95%        43         43.00       4.92104       4.93023       0.78720
1.05473       1.11247       0.87761       0.24595       0.79128     224.79070   47.83603
         90%        41         41.00       4.92429       4.82927       0.82102
0.97059       0.94205       0.81576       0.24299       0.82720     215.80488   38.62403
         80%        36         36.00       4.79946       4.75000       0.86032
0.78977       0.62373       0.68038       0.18477       0.86118     160.75000   22.45430
      75% Q3        34         34.00       4.81818       4.67647       0.88526
0.72427       0.52456       0.63107       0.18041       0.89027     155.44118   17.83516
         70%        32         32.00       4.85033       4.78125       0.90374
0.67053       0.44961       0.58902       0.16453       0.90504     149.46875   14.38756
         60%        27         27.00       4.67844       4.55556       0.93836
0.55447       0.30743       0.49382       0.15997       0.94253     134.66667   8.30069
       50% Median   23         23.00       4.88336       4.82609       0.95549
0.46827       0.21928       0.42309       0.11004       0.95677     113.30435   5.04347
         40%        18         18.00       4.80570       4.77778       0.97340
0.37881       0.14350       0.34681       0.09554       0.97707      97.11111   2.58294
         30%        14         14.00       4.61965       4.50000       0.97774
0.31270       0.09778       0.28869       0.08793       0.98232      61.50000   1.36894
      25% Q1        12         12.00       4.64520       4.50000       0.98040
0.28291       0.08004       0.26169       0.07944       0.98561      49.00000   0.96046
         20%         9          9.00       4.59693       4.44444       0.98956
0.23160       0.05364       0.21614       0.07654       0.99413      46.22222   0.48276
         10%         5          5.00       4.44316       4.40000       0.98790
0.16460       0.02709       0.15773       0.03656       0.99000      11.20000   0.13547
          5%         3          3.00       3.40530       3.33333       0.91523
0.13725       0.01884       0.13042       0.03823       0.97823       0.66667   0.05652
          4%         2          2.00       3.01353       3.00000        .
0.10212       0.01043       0.10122       0.03374        .            0.00000   0.02086
          3%         2          2.00       3.01353       3.00000        .
0.10212       0.01043       0.10122       0.03374        .            0.00000   0.02086
        2.5%         2          2.00       3.01353       3.00000        .
0.10212       0.01043       0.10122       0.03374        .            0.00000   0.02086
          2%         1          1.00       2.91231       3.00000        .
0.08769       0.00769       0.08769       0.02923        .            0.00000   0.00769
          1%         1          1.00       2.91231       3.00000        .
0.08769       0.00769       0.08769       0.02923        .            0.00000   0.00769
-------------------------------------------------------------------------------------
      97.78%        -1         44.00       4.87229       4.93182       0.76520
1.09527       1.19961       0.90821       0.25047       0.76735     224.79545 52.78285
      88.89%        -5         40.00       4.92252       4.87500       0.83688
0.93063       0.86607       0.78627       0.23244       0.84194     212.37500 34.64286
      77.78%       -10         35.00       4.78150       4.68571       0.87153
0.75561       0.57094       0.65491       0.18363       0.87371     155.54286 19.98301
      44.44%       -25         20.00       4.92297       4.90000       0.96712
0.40908       0.16735       0.37390       0.09652       0.97122     101.80000   3.34697

 ==============================================
 Percentage of Error Statistics Due To Outliers
 ==============================================

 % Outliers         % MAD     Lift(MAD)         % MSE     Lift(MSE)         % MAPE
N          Wgt N
-------------------------------------------------------------------------------------
 Lrn      1%         6.15          6.15         11.50         11.50          16.85
1           1.00
          2%         6.15          3.08         11.50          5.75          16.85
1           1.00
        2.5%        11.37          4.55         19.80          7.92          24.91
2           2.00
```

```
         3%        11.37          3.79         19.80          6.60          24.91
2         2.00
         4%        11.37          2.84         19.80          4.95          24.91
2         2.00
         5%        16.53          3.31         27.88          5.58          30.67
3         3.00
        10%        26.14          2.61         41.92          4.19          39.42
5         5.00
        20%        42.48          2.12         62.35          3.12          53.53
9         9.00
        25% Q1     53.04          2.12         73.67          2.95          61.60
12        12.00
        30%        58.40          1.95         78.04          2.60          66.36
14        14.00
        40%        68.69          1.72         86.08          2.15          74.14
18        18.00
        50% Median 79.00          1.58         92.59          1.85          82.60
23        23.00
        60%        85.34          1.42         95.67          1.59          88.80
27        27.00
        70%        91.65          1.31         98.10          1.40          93.73
32        32.00
        75% Q3     93.60          1.25         98.68          1.32          95.21
34        34.00
        80%        95.43          1.19         99.19          1.24          96.47
36        36.00
        90%        98.61          1.10         99.84          1.11          98.95
41        41.00
        95%        99.52          1.05         99.97          1.05          99.53
43        43.00
        96%        99.79          1.04         99.99          1.04          99.78
44        44.00
        97%        99.79          1.03         99.99          1.03          99.78
44        44.00
        97.5%      99.79          1.02         99.99          1.03          99.78
44        44.00
        98%       100.00          1.02        100.00          1.02         100.00
45        45.00
        99%       100.00          1.01        100.00          1.01         100.00
45        45.00
       100%       100.00          1.00        100.00          1.00         100.00
45        45.00
-----------------------------------------------------------------------------------
      2.22%         6.15          2.77         11.50          5.18          16.85
1         1.00
     11.11%        26.14          2.35         41.92          3.77          39.42
5         5.00
     22.22%        46.17          2.08         66.50          2.99          56.29
10        10.00
     55.56%        82.44          1.48         94.39          1.70          85.75
25        25.00
----------------------------------------------------------------------------------

 ==================================================
 Learn Sample Residual Fit Diagnostics - 7-BF Model
 ==================================================

             Mean          Min          Max        Wgt N
 --------------------------------------------------------
 Y        4.93333      1.00000     10.00000        45.00
 YHat     4.93333      1.32889      9.22812        45.00

--------- Predicted Response -------          ------ Standardized Residual -------

      N            W       Mean(Y)        Mean          Min           Max
StdDev          IQ1          IQ3
----------------------------------------------------------------------------------
```

```
       3        3.00       1.66667      1.85391      1.32889      2.30247
0.58378     -0.80811       0.60588
       3        3.00       3.33333      2.54829      2.41707      2.77585
0.94784     -0.36227       1.93191
       3        3.00       2.00000      2.93435      2.91231      2.94536
0.72097     -1.68976       0.07617
       3        3.00       3.33333      3.18442      3.11475      3.23917
0.40273     -0.20775       0.69546
       2        2.00       4.00000      3.52433      3.51422      3.53445
0.85982     -0.44665       1.27299
       2        2.00       4.00000      3.83928      3.76601      3.91256
0.80496     -0.66536       0.94456
       2        2.00       3.50000      4.16862      4.14840      4.18883
0.41674     -0.99751      -0.16402
       2        2.00       4.00000      4.39128      4.36569      4.41688
0.02223     -0.36210      -0.31764
       2        2.00       4.00000      4.63014      4.45903      4.80125
0.71998     -1.26733       0.17264
       2        2.00       7.00000      4.85443      4.80413      4.90473
0.04369      1.81997       1.90735
       2        2.00       4.50000      4.95316      4.91103      4.99529
1.33951     -1.73312       0.94589
       2        2.00       6.00000      5.39417      5.36038      5.42796
0.83926     -0.31303       1.36548
       2        2.00       5.50000      5.59175      5.58747      5.59602
0.43802     -0.51771       0.35833
       2        2.00       5.00000      6.02313      5.92569      6.12058
0.08464     -0.97334      -0.80406
       2        2.00       7.50000      6.37935      6.36063      6.39807
0.41805      0.55536       1.39145
       2        2.00       6.00000      6.55450      6.51572      6.59328
0.90229     -1.38394       0.42065
       2        2.00       7.00000      6.97422      6.86483      7.08360
0.96362     -0.94122       0.98602
       2        2.00       7.50000      7.30379      7.19863      7.40894
0.34297     -0.17253       0.51340
       2        2.00       6.00000      7.70437      7.61931      7.78942
0.79473     -2.27516      -0.68570
       3        3.00       9.33333      8.62136      8.18358      9.22812
0.73201     -0.19815       1.57776
-----------------------------------------------------------------------------------
     45       45.00

 Grove file created: C:\Users\ayseo\AppData\Local\Temp\vi4_00818.grv: 102 kb,
 78% compression

 Grove file created containing:
      1 Mars model

 Import processed data cache : 00:00:00
 MARS model building         : 00:00:02
 Total                       : 00:00:02
>REM
>
```

MARS model for Y_4 in *MQ1*

```
The KEEP list has 6 variables.

 Salford Predictive Modeler(R) software suite: MARS(R) version 8.3.2.001

 Data in cache:
 N variables: 16
 N learn records: 45

 The set of model variables appears to have changed.
```

```
Checking if they are a subset of the cached data with
consistent coding (continuous, categorical).

The current set of model variables is found
to be a subset of those in the data cache.

                        N
-------------------------
         Learn         45
          Test          0
       Holdout          0
-------------------------
         Total         45

============
MARS Results
============

=================
Distribution of Y
=================

-------------------------
N                      45
Sum(Weights)        45.00
Mean              4.68889
Median            4.00000
Range             8.00000
Sum             211.00000
Cond. Mean        4.68889
Std Dev           2.25451
N = 0                   0
N != 0                 45
-------------------------
MSE               4.96988
RMSE              2.22932
MAD               1.75556
MAPE              0.43090
SSY             223.64444
SSE             223.64444
-------------------------
     Minimum      1.00000
          1%      1.00000
          2%      1.00000
        2.5%      2.00000
          3%      2.00000
          4%      2.00000
          5%      2.00000
         10%      2.00000
         20%      3.00000
      25% Q1      3.00000
         30%      3.00000
         40%      4.00000
-------------------------
  50% Median      4.00000
-------------------------
         60%      5.00000
         70%      5.00000
      75% Q3      6.00000
         80%      7.50000
         90%      8.00000
         95%      9.00000
         96%      9.00000
         97%      9.00000
       97.5%      9.00000
         98%      9.00000
         99%      9.00000
     Maximum      9.00000
```

```
================================
Forward Stepwise Knot Placement
================================

BasFn(s)      GCV  IndBsFns EfPrms Variable                    Knot   Parent  BsF
------------------------------------------------------------------------------------
    0        5.19835    0.0    1.0                                                2
    1        5.70302    2.0    6.0 X9                          2.50000            4
    3        5.92013    4.0   11.0 X2                          7.70000            6
    5        5.10899    6.0   16.0 X6                          3.60000 X2         8
    7        6.23263    8.0   21.0 X12                         3.10000           10
    9        8.43633   10.0   26.0 X6                          3.30000 X9        12
   11       12.12417   12.0   31.0 X14                         4.00000 X9        14
   13       21.24830   14.0   36.0 X12                         4.70000 X9         2
   15       53.61680   15.0   40.0 X5                          1.00000 X12       14

==================================================
Final Model (After Backward Stepwise Elimination)
==================================================

Basis Fun  Coefficient Variable                   Knot    Parent
-----------------------------------------------------------------
        0       5.71241
        1      -1.12175 X9                        2.50000
        4      -0.68640 X2                        7.70000
        6       0.60375 X6                        3.60000   X2
        7       0.88840 X12                       3.10000
       10      -2.35139 X6                        3.30000   X9
       11      -1.18374 X14                       4.00000   X9
       13      -2.77556 X12                       4.70000   X9
       15       0.28329 X5                        1.00000   X12

Piecewise Linear GCV = 3.48382, #efprms = 21.80000

========================================
ANOVA Decomposition on 8 Basis Functions
========================================

 fun     std. dev.         -gcv #bsfns  #efprms variable
-------------------------------------------------------
   1      1.53269      6.55176    1     2.60000   X9
   2      1.74336      8.45811    1     2.60000   X2
   3      1.35394      4.69944    1     2.60000   X12
   4      2.31112      9.85499    1     2.60000   X2
                                                  X6
   5      1.35504      5.68521    1     2.60000   X6
                                                  X9
   6      0.84748      4.82015    1     2.60000   X9
                                                  X14
   7      1.30260      5.17453    1     2.60000   X9
                                                  X12
   8      0.78692      3.89110    1     2.60000   X5
                                                  X9
                                                  X12

===================
Variable Importance
===================

Variable                                Importance         -gcv
---------------------------------------------------------------
X2                                       100.00000      9.22249
X6                                        90.20428      8.15327
X9                                        62.50936      5.72616
X14                                       48.25612      4.82015
X12                                       36.86394      4.26367
X5                                        26.64059      3.89110
```

```
================================
MARS Regression: Training Data
================================

W: 45.00                                   R-SQUARED: 0.81368
MEAN DEP VAR: 4.68889                  ADJ R-SQUARED: 0.77228
                  UNCENTERED R-SQUARED = R-0 SQUARED: 0.96565

   Parameter                    Estimate         S.E.       T-Value       P-Value
-----------------------------------------------------------------------------
Constant                 |       5.71241      0.40442      14.12481       0.00000
Basis Function 1         |      -1.12175      0.16237      -6.90851       0.00000
Basis Function 4         |      -0.68640      0.08084      -8.49056       0.00000
Basis Function 6         |       0.60375      0.06366       9.48372       0.00000
Basis Function 7         |       0.88840      0.18113       4.90470       0.00002
Basis Function 10        |      -2.35139      0.38839      -6.05423       0.00000
Basis Function 11        |      -1.18374      0.23396      -5.05952       0.00001
Basis Function 13        |      -2.77557      0.50567      -5.48885       0.00000
Basis Function 15        |       0.28329      0.07646       3.70486       0.00071
-----------------------------------------------------------------------------
F-STATISTIC =  19.65194                     S.E. OF REGRESSION =  1.07587
    P-VALUE =  0.00000                 RESIDUAL SUM OF SQUARES =  41.66952
  [MDF,NDF] = [ 8, 36 ]              REGRESSION SUM OF SQUARES =  181.97492
-----------------------------------------------------------------------------

===============
Basis Functions
===============

BF1 = max( 0, X9 - 2.5);
BF2 = max( 0, 2.5 - X9);
BF4 = max( 0, 7.7 - X2);
BF6 = max( 0, 3.6 - X6) * BF4;
BF7 = max( 0, X12 - 3.1);
BF10 = max( 0, 3.3 - X6) * BF2;
BF11 = max( 0, X14 - 4) * BF2;
BF13 = max( 0, X12 - 4.7) * BF2;
BF14 = max( 0, 4.7 - X12) * BF2;
BF15 = max( 0, X5 - 1) * BF14;

Y = 5.71241 - 1.12175 * BF1 - 0.686401 * BF4 + 0.603748 * BF6
            + 0.888396 * BF7 - 2.35139 * BF10 - 1.18374 * BF11
            - 2.77556 * BF13 + 0.28329 * BF15;

MODEL Y = BF1 BF4 BF6 BF7 BF10 BF11 BF13 BF15;

=============
Selector Info
=============

DOF Penalty = 3

BasFn  TotVar DirVar       EffPar          GCV    Learn MSE        Adj MSE   RMSE
----------------------------------------------------------------------------------
    15      6      6     40.00000     53.61681      0.66194        0.42658  0.81359
    14      6      6     37.40000     23.43611      0.66848        0.44565  0.81761
    13      6      6     34.80000     13.06213      0.67110        0.46232  0.81921
    12      6      6     32.20000      8.38655      0.67854        0.48252  0.82374
    11      6      6     29.60000      5.93842      0.69548        0.51002  0.83396
    10      6      6     27.00000      4.65513      0.74482        0.56275  0.86303
     9      6      6     24.40000      3.84074      0.80487        0.62601  0.89714
 **  8      6      6     21.80000      3.48382      0.92599        0.74079  0.96228
     7      5      5     19.20000      3.89110      1.27905        1.05166  1.13095
     6      5      5     16.60000      4.10731      1.63595        1.38147  1.27904
     5      4      4     14.00000      4.26367      2.02340        1.75362  1.42246
     4      3      3     11.40000      4.62616      2.57914        2.29257  1.60597
     3      3      3      8.80000      4.71924      3.05397        2.78250  1.74756
```

2	2	2	6.20000	5.01003	3.72459	3.47629	1.92992
1	2	2	3.60000	5.33227	4.51323	4.31264	2.12444
0	0	0	1.00000	5.19835	4.96988	.	2.22932

Regression Performance Summary

Sample	Joint N	Wgt Joint N	Mean(Score)	Mean(Target)	R-Sq	RMSE	MSE	MAD	MAPE	Norm R-Sq	SSY	SSE
Lrn	45	45.00	4.68889	4.68889	0.81368	0.96228	0.92599	0.82336	0.22874	0.81368	223.64444	41.66952

Performance By Abs(Deviation) Outlier Trimming

Percentile	Joint N	Wgt Joint N	Mean(Score)	Mean(Target)	R-Sq	RMSE	MSE	MAD	MAPE	Norm R-Sq	SSY	SSE
Lrn 100%	45	45.00	4.68889	4.68889	0.81368	0.96228	0.92599	0.82336	0.22874	0.81368	223.64444	41.66952
99%	45	45.00	4.68889	4.68889	0.81368	0.96228	0.92599	0.82336	0.22874	0.81368	223.64444	41.66952
98%	45	45.00	4.68889	4.68889	0.81368	0.96228	0.92599	0.82336	0.22874	0.81368	223.64444	41.66952
97.5%	44	44.00	4.72936	4.68182	0.83317	0.92064	0.84757	0.79453	0.22443	0.83398	223.54545	37.29317
97%	44	44.00	4.72936	4.68182	0.83317	0.92064	0.84757	0.79453	0.22443	0.83398	223.54545	37.29317
96%	44	44.00	4.72936	4.68182	0.83317	0.92064	0.84757	0.79453	0.22443	0.83398	223.54545	37.29317
95%	43	43.00	4.72737	4.72093	0.84592	0.88920	0.79067	0.77080	0.21558	0.84632	220.65116	33.99892
90%	41	41.00	4.75054	4.82927	0.86723	0.82428	0.67943	0.72291	0.19045	0.86857	209.80488	27.85664
80%	36	36.00	4.61416	4.66667	0.88830	0.70458	0.49643	0.62732	0.16981	0.89073	160.00000	17.87160
75% Q3	34	34.00	4.64107	4.76471	0.90011	0.66853	0.44693	0.59619	0.15425	0.90478	152.11765	15.19566
70%	32	32.00	4.77672	4.90625	0.90986	0.62960	0.39640	0.56344	0.13487	0.91579	140.71875	12.68476
60%	27	27.00	4.51044	4.55556	0.92497	0.52891	0.27975	0.48088	0.12907	0.92603	100.66667	7.55325
50% Median	23	23.00	4.40483	4.52174	0.93854	.47304	0.22377	0.42986	0.11808	0.94229	83.73913	5.14666
40%	18	18.00	3.99469	4.11111	0.95233	0.39788	0.15831	0.36115	0.11749	0.95804	59.77778	2.84949
30%	14	14.00	4.21006	4.35714	0.96641	0.33658	0.11328	0.30419	0.07938	0.97284	47.21429	1.58598
25% Q1	12	12.00	4.41024	4.50000	0.97548	0.30321	0.09193	0.27306	0.06862	0.97839	45.00000	1.10321
20%	9	9.00	4.15422	4.22222	0.98619	0.24003	0.05762	0.21701	0.06243	0.98740	37.55556	0.51854
10%	5	5.00	3.82072	3.80000	0.98924	0.13758	0.01893	0.13201	0.03743	0.98949	8.80000	0.09465
5%	3	3.00	3.03177	3.00000	0.97879	0.11892	0.01414	0.11232	0.03984	0.99951	2.00000	0.04243
4%	2	2.00	2.46999	2.50000	0.96341	0.09565	0.00915	0.09082	0.04034	1.00000	0.50000	0.01830
3%	2	2.00	2.46999	2.50000	0.96341	0.09565	0.00915	0.09082	0.04034	1.00000	0.50000	0.01830
2.5%	2	2.00	2.46999	2.50000	0.96341	0.09565	0.00915	0.09082	0.04034	1.00000	0.50000	0.01830

```
         2%          1          1.00       3.06081      3.00000           .
0.06081      0.00370      0.06081       0.02027           .          0.00000     0.00370
         1%          1          1.00       3.06081      3.00000           .
0.06081      0.00370      0.06081       0.02027           .          0.00000     0.00370
 -----------------------------------------------------------------------------------
     97.78%         -1         44.00       4.72936      4.68182      0.83317
0.92064      0.84757      0.79453       0.22443      0.83398     223.54545    37.29317
     88.89%         -5         40.00       4.83227      4.87500      0.87619
0.79923      0.63876      0.70302       0.18256      0.87799     206.37500    25.55057
     77.78%        -10         35.00       4.65565      4.74286      0.89179
0.68706      0.47205      0.61205       0.15806      0.89449     152.68571    16.52169
     44.44%        -25         20.00       4.18063      4.35000      0.94777
0.42923      0.18424      0.38963       0.11576      0.95974      70.55000     3.68481

 ===============================================
 Percentage of Error Statistics Due To Outliers
 ===============================================

 % Outliers         % MAD    Lift(MAD)          % MSE     Lift(MSE)         % MAPE
N         Wgt N
----------------------------------------------------------------------------------
 Lrn      1%          5.65          5.65          10.50         10.50           8.55
1          1.00
          2%          5.65          2.82          10.50          5.25           8.55
1          1.00
        2.5%         10.54          4.22          18.41          7.36          15.23
2          2.00
          3%         10.54          3.51          18.41          6.14          15.23
2          2.00
          4%         10.54          2.64          18.41          4.60          15.23
2          2.00
          5%         15.29          3.06          25.84          5.17          21.10
3          3.00
         10%         24.10          2.41          38.68          3.87          32.40
5          5.00
         20%         39.05          1.95          57.11          2.86          52.25
9          9.00
         25% Q1      48.39          1.94          66.70          2.67          61.42
12         12.00
         30%         54.20          1.81          72.25          2.41          66.45
14         14.00
         40%         64.96          1.62          81.87          2.05          73.90
18         18.00
         50% Median  75.22          1.50          88.84          1.78          81.67
23         23.00
         60%         82.45          1.37          93.16          1.55          86.82
27         27.00
         70%         89.88          1.28          96.81          1.38          92.29
32         32.00
         75% Q3      92.41          1.23          97.87          1.30          94.09
34         34.00
         80%         94.73          1.18          98.76          1.23          95.82
36         36.00
         90%         98.67          1.10          99.84          1.11          98.82
41         41.00
         95%         99.51          1.05          99.96          1.05          99.55
43         43.00
         96%         99.84          1.04          99.99          1.04          99.80
44         44.00
         97%         99.84          1.03          99.99          1.03          99.80
44         44.00
         97.5%       99.84          1.02          99.99          1.03          99.80
44         44.00
         98%        100.00          1.02         100.00          1.02         100.00
45         45.00
         99%        100.00          1.01         100.00          1.01         100.00
45         45.00
```

```
      100%       100.00         1.00       100.00         1.00       100.00
45        45.00
---------------------------------------------------------------------------------
     2.22%         5.65         2.54        10.50         4.73         8.55
1        1.00
    11.11%        24.10         2.17        38.68         3.48        32.40
5        5.00
    22.22%        42.18         1.90        60.35         2.72        55.97
10       10.00
    55.56%        78.97         1.42        91.16         1.64        84.31
25       25.00
---------------------------------------------------------------------------------

 ==================================================
 Learn Sample Residual Fit Diagnostics - 8-BF Model
 ==================================================

             Mean          Min          Max         Wgt N
 -------------------------------------------------------
 Y         4.68889      1.00000      9.00000        45.00
 YHat      4.68889      1.48142      9.48429        45.00

--------- Predicted Response -------           ------ Standardized Residual -------

      N             W       Mean(Y)         Mean          Min          Max
StdDev           IQ1          IQ3
 ---------------------------------------------------------------------------------
      3          3.00      2.00000      1.55638      1.48142      1.61434
0.88853     -0.59585      1.57810
      3          3.00      2.66667      2.07382      1.85063      2.49165
0.44075      0.12556      1.19442
      3          3.00      4.00000      2.96998      2.90803      3.06081
0.91356     -0.06320      2.17397
      3          3.00      2.00000      3.20912      3.09080      3.37469
0.12535     -1.42857     -1.13356
      3          3.00      3.66667      3.51288      3.48283      3.52947
0.50241     -0.55023      0.53744
      2          2.00      3.50000      3.73142      3.70501      3.75783
0.54704     -0.78753      0.30655
      2          2.00      2.50000      3.79007      3.75992      3.82021
0.48827     -1.82890     -0.85236
      2          2.00      4.00000      4.16050      4.15534      4.16567
0.00537     -0.17216     -0.16142
      2          2.00      4.50000      4.29543      4.28874      4.30212
0.51265     -0.30006      0.72523
      2          2.00      4.50000      4.40359      4.37795      4.42923
0.54624     -0.44605      0.64643
      2          2.00      4.50000      4.61479      4.53718      4.69241
0.60025     -0.71955      0.48096
      2          2.00      3.00000      4.77999      4.74498      4.81501
0.03639     -1.88614     -1.81337
      2          2.00      5.00000      5.24188      5.15153      5.33223
0.94530     -1.19667      0.69394
      2          2.00      5.50000      5.77424      5.70588      5.84260
0.44856     -0.73355      0.16357
      2          2.00      6.00000      6.21785      6.05972      6.37598
0.87487     -1.10125      0.64848
      2          2.00      7.50000      6.61176      6.56846      6.65506
0.47460      0.44845      1.39766
      2          2.00      8.00000      6.95359      6.94205      6.96513
0.01199      1.07543      1.09942
      2          2.00      8.50000      7.42685      7.18682      7.66688
0.27016      0.84506      1.38537
      2          2.00      9.00000      8.42072      8.16513      8.67631
0.26561      0.33637      0.86759
      2          2.00      8.00000      9.09406      8.70383      9.48429
0.40552     -1.54247     -0.73142
---------------------------------------------------------------------------------
```

```
    45        45.00

 Grove file created: C:\Users\ayseo\AppData\Local\Temp\vi4_00919.grv: 76 kb,
 82% compression

 Grove file created containing:
      1 Mars model

 Import processed data cache : 00:00:00
 MARS model building         : 00:00:02
 Total                       : 00:00:02
>REM
```

MARS model for Y_1 in *MQ2*

```
The KEEP list has 3 variables.

 Salford Predictive Modeler(R) software suite: MARS(R) version 8.3.2.001

 Data in cache:
 N variables: 36
 N learn records: 20

 The set of model variables appears to have changed.
 Checking if they are a subset of the cached data with
 consistent coding (continuous, categorical).

 The current set of model variables is found
 to be a subset of those in the data cache.

                         N
 -------------------------
          Learn         20
           Test          0
        Holdout          0
 -------------------------
          Total         20

 ============
 MARS Results
 ============

 =================
 Distribution of Y
 =================

 -------------------------
 N                      20
 Sum(Weights)        20.00
 Mean              3.95000
 Median            4.00000
 Range             5.00000
 Sum              79.00000
 Cond. Mean        3.95000
 Std Dev           1.63755
 N = 0                   0
 N != 0                 20
 -------------------------
 MSE               2.54750
 RMSE              1.59609
 MAD               1.35000
 MAPE              0.53333
 SSY              50.95000
 SSE              50.95000
 -------------------------
      Minimum      1.00000
```

```
          1%       1.00000
          2%       1.00000
        2.5%       1.00000
          3%       1.00000
          4%       1.00000
          5%       1.50000
         10%       2.00000
         20%       2.00000
      25% Q1       2.00000
         30%       2.50000
         40%       4.00000
-------------------------
  50% Median       4.00000
-------------------------
         60%       5.00000
         70%       5.00000
      75% Q3       5.00000
         80%       5.50000
         90%       6.00000
         95%       6.00000
         96%       6.00000
         97%       6.00000
       97.5%       6.00000
         98%       6.00000
         99%       6.00000
     Maximum       6.00000

===============================
Forward Stepwise Knot Placement
===============================

BasFn(s)      GCV  IndBsFns EfPrms Variable                    Knot   Parent  BsF
--------------------------------------------------------------------------------
    0       2.82271    0.0    1.0
    1       2.62448    1.0    5.0 X9                           1.00000
    2       3.16554    2.0    9.0 X16                          1.00000 X9    1
    3       5.13794    3.0   13.0 X6                           1.00000
    4      18.57221    4.0   17.0 X6                           1.00000 X16   2


=================================================
Final Model (After Backward Stepwise Elimination)
=================================================

Basis Fun  Coefficient Variable                    Knot   Parent
----------------------------------------------------------------
       0      4.73050
       3     -0.70654 X6                           1.00000
       4      0.06181 X6                           1.00000 X16

Piecewise Linear GCV = 1.49888, #efprms = 9.00000


========================================
ANOVA Decomposition on 2 Basis Functions
========================================

 fun    std. dev.         -gcv #bsfns  #efprms variable
-------------------------------------------------------
   1      1.19487      2.78542   1      4.00000 X6
   2      1.55096      4.14095   1      4.00000 X6
                                                X9
                                                X16

===================
Variable Importance
===================
```

```
Variable                                   Importance        -gcv
---------------------------------------------------------------
X16                                         100.00000      4.14095
X9                                          100.00000      4.14095
X6                                           70.78565      2.82271

==============================
MARS Regression: Training Data
==============================

W: 20.00                                   R-SQUARED: 0.82202
MEAN DEP VAR: 3.95000                  ADJ R-SQUARED: 0.80108
                  UNCENTERED R-SQUARED = R-0 SQUARED: 0.97502

   Parameter                  Estimate        S.E.      T-Value      P-Value
----------------------------------------------------------------------------
Constant                 |     4.73050     0.34974     13.52580      0.00000
Basis Function 3         |    -0.70654     0.10935     -6.46104      0.00001
Basis Function 4         |     0.06181     0.00737      8.38650      0.00000
----------------------------------------------------------------------------
F-STATISTIC =  39.25757                S.E. OF REGRESSION =  0.73036
    P-VALUE =  0.00000             RESIDUAL SUM OF SQUARES =  9.06820
  [MDF,NDF] = [ 2, 17 ]          REGRESSION SUM OF SQUARES =  41.88180
----------------------------------------------------------------------------

===============
Basis Functions
===============

BF1 = max( 0, X9 - 1);
BF2 = max( 0, X16 - 1) * BF1;
BF3 = max( 0, X6 - 1);
BF4 = max( 0, X6 - 1) * BF2;

Y = 4.7305 - 0.706543 * BF3 + 0.0618139 * BF4;

MODEL Y = BF3 BF4;

=============
Selector Info
=============

DOF Penalty = 3

BasFn  TotVar DirVar        EffPar          GCV     Learn MSE       Adj MSE      RMSE
---------------------------------------------------------------------------------
     4      3      3      17.00000     18.57221       0.41787       0.31341   0.64643
     3      3      3      13.00000      3.44908       0.42251       0.33801   0.65001
 **  2      3      3       9.00000      1.49888       0.45341       0.38540   0.67336
     1      3      3       5.00000      2.78542       1.56680       1.41012   1.25172
     0      0      0       1.00000      2.82271       2.54750         .       1.59609
---------------------------------------------------------------------------------

==============================
Regression Performance Summary
==============================

 Sample       Joint N  Wgt Joint N  Mean(Score) Mean(Target)          R-Sq
RMSE           MSE           MAD          MAPE     Norm R-Sq           SSY          SSE
---------------------------------------------------------------------------------
 Lrn               20        20.00      3.95000      3.95000       0.82202
0.67336       0.45341      0.50338      0.15915      0.82202      50.95000      9.06820


==============================================
Performance By Abs(Deviation) Outlier Trimming
==============================================
```

Percentile	Joint N	Wgt Joint N	Mean(Score)	Mean(Target)	R-Sq	RMSE	MSE	MAD	MAPE	Norm R-Sq	SSY	SSE
Lrn 100%	20	20.00	3.95000	3.95000	0.82202	0.67336	0.45341	0.50338	0.15915	0.82202	50.95000	9.06820
99%	20	20.00	3.95000	3.95000	0.82202	0.67336	0.45341	0.50338	0.15915	0.82202	50.95000	9.06820
98%	20	20.00	3.95000	3.95000	0.82202	0.67336	0.45341	0.50338	0.15915	0.82202	50.95000	9.06820
97.5%	20	20.00	3.95000	3.95000	0.82202	0.67336	0.45341	0.50338	0.15915	0.82202	50.95000	9.06820
97%	20	20.00	3.95000	3.95000	0.82202	0.67336	0.45341	0.50338	0.15915	0.82202	50.95000	9.06820
96%	20	20.00	3.95000	3.95000	0.82202	0.67336	0.45341	0.50338	0.15915	0.82202	50.95000	9.06820
95%	19	19.00	3.94425	3.84211	0.88606	0.52822	0.27902	0.42772	0.15050	0.89034	46.52632	5.30140
90%	18	18.00	3.94769	3.77778	0.91017	0.47448	0.22513	0.38939	0.14644	0.92170	45.11111	4.05229
80%	16	16.00	3.95586	3.87500	0.93023	0.39483	0.15589	0.32777	0.09747	0.93325	35.75000	2.49419
75% Q3	15	15.00	3.90421	3.86667	0.94513	0.36153	0.13070	0.30092	0.09179	0.94576	35.73333	1.96056
70%	14	14.00	3.78215	3.78571	0.95388	0.33643	0.11319	0.27862	0.08959	0.95540	34.35714	1.58460
60%	12	12.00	3.81839	3.91667	0.96951	0.28027	0.07855	0.23095	0.06827	0.97414	30.91667	0.94264
50% Median	10	10.00	3.87891	3.90000	0.97932	0.21762	0.04736	0.18029	0.04935	0.98263	22.90000	0.47358
40%	8	8.00	3.85703	3.87500	0.99171	0.14708	0.02163	0.12882	0.03650	0.99312	20.87500	0.17307
30%	6	6.00	3.38342	3.33333	0.99445	0.11110	0.01234	0.09771	0.03502	0.99558	13.33333	0.07406
25% Q1	5	5.00	3.62979	3.60000	0.99544	0.10108	0.01022	0.08694	0.02686	0.99612	11.20000	0.05109
20%	4	4.00	3.00152	3.00000	0.99233	0.08756	0.00767	0.07295	0.02763	0.99415	4.00000	0.03067
10%	2	2.00	2.99936	3.00000	0.99939	0.02460	0.00061	0.02459	0.00930	1.00000	2.00000	0.00121
5%	1	1.00	4.02395	4.00000	.	0.02395	0.00057	0.02395	0.00599	.	0.00000	0.00057
4%	1	1.00	4.02395	4.00000	.	0.02395	0.00057	0.02395	0.00599	.	0.00000	0.00057
3%	1	1.00	4.02395	4.00000	.	0.02395	0.00057	0.02395	0.00599	.	0.00000	0.00057
2.5%	1	1.00	4.02395	4.00000	.	0.02395	0.00057	0.02395	0.00599	.	0.00000	0.00057
2%	1	1.00	4.02395	4.00000	.	0.02395	0.00057	0.02395	0.00599	.	0.00000	0.00057
1%	1	1.00	4.02395	4.00000	.	0.02395	0.00057	0.02395	0.00599	.	0.00000	0.00057
95.00%	-1	19.00	3.94425	3.84211	0.88606	0.52822	0.27902	0.42772	0.15050	0.89034	46.52632	5.30140
75.00%	-5	15.00	3.90421	3.86667	0.94513	0.36153	0.13070	0.30092	0.09179	0.94576	35.73333	1.96056
50.00%	-10	10.00	3.87891	3.90000	0.97932	0.21762	0.04736	0.18029	0.04935	0.98263	22.90000	0.47358

Percentage of Error Statistics Due To Outliers

% Outliers	% MAD	Lift(MAD)	% MSE	Lift(MSE)	% MAPE	N	Wgt N
Lrn 1%	19.28	19.28	41.54	41.54	28.41	1	1.00

```
         2%        19.28          9.64        41.54        20.77        28.41
1          1.00
       2.5%        19.28          7.71        41.54        16.62        28.41
1          1.00
         3%        19.28          6.43        41.54        13.85        28.41
1          1.00
         4%        19.28          4.82        41.54        10.38        28.41
1          1.00
         5%        19.28          3.86        41.54         8.31        28.41
1          1.00
        10%        30.38          3.04        55.31         5.53        38.57
2          2.00
        20%        47.91          2.40        72.50         3.62        55.92
4          4.00
        25% Q1     55.16          2.21        78.38         3.14        62.94
5          5.00
        30%        61.25          2.04        82.53         2.75        68.68
6          6.00
        40%        72.47          1.81        89.60         2.24        78.16
8          8.00
        50% Median 82.09          1.64        94.78         1.90        85.70
10        10.00
        60%        89.76          1.50        98.09         1.63        90.83
12        12.00
        70%        94.18          1.35        99.18         1.42        95.17
14        14.00
        75% Q3     95.68          1.28        99.44         1.33        96.64
15        15.00
        80%        97.10          1.21        99.66         1.25        97.74
16        16.00
        90%        99.51          1.11        99.99         1.11        99.42
18        18.00
        95%        99.76          1.05        99.99         1.05        99.81
19        19.00
        96%       100.00          1.04       100.00         1.04       100.00
20        20.00
        97%       100.00          1.03       100.00         1.03       100.00
20        20.00
      97.5%       100.00          1.03       100.00         1.03       100.00
20        20.00
        98%       100.00          1.02       100.00         1.02       100.00
20        20.00
        99%       100.00          1.01       100.00         1.01       100.00
20        20.00
       100%       100.00          1.00       100.00         1.00       100.00
20        20.00
-----------------------------------------------------------------------------------
      5.00%        19.28          3.86        41.54         8.31        28.41
1          1.00
     25.00%        55.16          2.21        78.38         3.14        62.94
5          5.00
     50.00%        82.09          1.64        94.78         1.90        85.70
10        10.00
 -----------------------------------------------------------------------------------

 ===================================================
 Learn Sample Residual Fit Diagnostics - 2-BF Model
 ===================================================


 Grove file created: C:\Users\ayseo\AppData\Local\Temp\vejk_00073.grv: 20 kb,
 84% compression

 Grove file created containing:
      1 Mars model

 Import processed data cache : 00:00:00
 MARS model building         : 00:00:00
```

```
 Total                        : 00:00:00
>REM
```

MARS model for Y_2 in *MQ2*

```
 ===========
 MARS Results
 ============

 ==================
 Distribution of Y2
 ==================

 -------------------------
 N                      20
 Sum(Weights)        20.00
 Mean              4.60000
 Median            5.00000
 Range             8.00000
 Sum              92.00000
 Cond. Mean        4.60000
 Std Dev           2.76063
 N = 0                   0
 N != 0                 20
 -------------------------
 MSE               7.24000
 RMSE              2.69072
 MAD               2.30000
 MAPE              1.14901
 SSY             144.80000
 SSE             144.80000
 -------------------------
      Minimum      1.00000
           1%      1.00000
           2%      1.00000
         2.5%      1.00000
           3%      1.00000
           4%      1.00000
           5%      1.00000
          10%      1.00000
          20%      1.50000
      25% Q1       2.00000
          30%      2.50000
          40%      3.50000
 -------------------------
   50% Median      5.00000
 -------------------------
          60%      5.00000
          70%      6.50000
      75% Q3       7.50000
          80%      8.00000
          90%      8.00000
          95%      8.50000
          96%      9.00000
          97%      9.00000
        97.5%      9.00000
          98%      9.00000
          99%      9.00000
      Maximum      9.00000

 ===============================
 Forward Stepwise Knot Placement
 ===============================

 BasFn(s)      GCV  IndBsFns EfPrms Variable                   Knot Parent   BsF
-----------------------------------------------------------------------------------
```

```
    0        8.02216    0.0    1.0
    1        7.72662    1.0     4.0 X35                       1.00000
    2        7.23759    2.0     7.0 X3                        1.00000
    3        5.73357    3.0    10.0 X7                        2.00000
    4        7.12396    4.0    13.0 X5                        1.00000
    5       15.65080    5.0    16.0 X15                       1.00000


================================================
Final Model (After Backward Stepwise Elimination)
================================================

Basis Fun  Coefficient Variable                   Knot  Parent
----------------------------------------------------------------
        0      1.30726
        1      1.16561 X35                        1.00000
        2     -0.78861 X3                         1.00000
        3      0.92635 X7                         2.00000

Piecewise Linear GCV = 5.73357, #efprms = 10.00001


=======================================
ANOVA Decomposition on 3 Basis Functions
=======================================

 fun    std. dev.        -gcv #bsfns  #efprms variable
------------------------------------------------------
   1      2.10053     12.62788   1      3.00000 X35
   2      1.56733      9.02455   1      3.00000 X3
   3      1.35513      7.23759   1      3.00000 X7

===================
Variable Importance
===================

Variable                                Importance         -gcv
--------------------------------------------------------------
X35                                      100.00000     12.62788
X3                                        69.09030      9.02455
X7                                        46.70698      7.23759
X16                                        0.00000      5.73357
X15                                        0.00000      5.73357
X14                                        0.00000      5.73357
X13                                        0.00000      5.73357
X12                                        0.00000      5.73357
X11                                        0.00000      5.73357
X10                                        0.00000      5.73357
X9                                         0.00000      5.73357
X8                                         0.00000      5.73357
X6                                         0.00000      5.73357
X5                                         0.00000      5.73357
X4                                         0.00000      5.73357
X2                                         0.00000      5.73357
X17                                        0.00000      5.73357
X18                                        0.00000      5.73357
X19                                        0.00000      5.73357
X34                                        0.00000      5.73357
X33                                        0.00000      5.73357
X32                                        0.00000      5.73357
X31                                        0.00000      5.73357
X30                                        0.00000      5.73357
X29                                        0.00000      5.73357
X28                                        0.00000      5.73357
X27                                        0.00000      5.73357
X26                                        0.00000      5.73357
X20                                        0.00000      5.73357
X21                                        0.00000      5.73357
```

```
X22                                       0.00000      5.73357
X23                                       0.00000      5.73357
X25                                       0.00000      5.73357
X24                                       0.00000      5.73357
X1                                        0.00000      5.73357

==============================
MARS Regression: Training Data
==============================

W: 20.00                                  R-SQUARED: 0.80202
MEAN DEP VAR: 4.60000                 ADJ R-SQUARED: 0.76490
                  UNCENTERED R-SQUARED = R-0 SQUARED: 0.94953

   Parameter                 Estimate         S.E.       T-Value      P-Value
-----------------------------------------------------------------------------
Constant               |      1.30726      0.89010       1.46866      0.16131
Basis Function 1       |      1.16562      0.17662       6.59956      0.00001
Basis Function 2       |     -0.78861      0.15302      -5.15370      0.00010
Basis Function 3       |      0.92635      0.21754       4.25830      0.00060
-----------------------------------------------------------------------------
F-STATISTIC =  21.60514                  S.E. OF REGRESSION =  1.33856
    P-VALUE =  0.00001               RESIDUAL SUM OF SQUARES =  28.66780
  [MDF,NDF] = [ 3, 16 ]            REGRESSION SUM OF SQUARES =  116.13220
-----------------------------------------------------------------------------

===============
Basis Functions
===============

BF1 = max( 0, X35 - 1);
BF2 = max( 0, X3 - 1);
BF3 = max( 0, X7 - 2);

Y = 1.30726 + 1.16561 * BF1 - 0.78861 * BF2 + 0.926349 * BF3;

MODEL Y2 = BF1 BF2 BF3;


=============
Selector Info
=============

DOF Penalty = 3

BasFn  TotVar DirVar       EffPar          GCV     Learn MSE      Adj MSE     RMSE
-----------------------------------------------------------------------------------
      5      5      5     16.00001     15.65079       0.62603       0.43822  0.79122
      4      4      4     13.00001      7.12396       0.87268       0.65451  0.93417
  **  3      3      3     10.00001      5.73357       1.43339       1.14671  1.19724
      2      2      2      7.00000      7.23759       3.05788       2.59920  1.74868
      1      1      1      4.00000      7.72662       4.94503       4.45053  2.22374
      0      0      0      1.00000      8.02216       7.24000        .        2.69072
-----------------------------------------------------------------------------------

==============================
Regression Performance Summary
==============================

Sample       Joint N  Wgt Joint N  Mean(Score) Mean(Target)         R-Sq
RMSE           MSE           MAD         MAPE     Norm R-Sq          SSY          SSE
-----------------------------------------------------------------------------------
Lrn             20        20.00      4.60000      4.60000      0.80202
1.19724      1.43339      0.91051      0.29163      0.80202    144.80000     28.66780

==============================================
Performance By Abs(Deviation) Outlier Trimming
==============================================
```

Percentile	Joint N	Wgt Joint N	Mean(Score)	Mean(Target)	R-Sq	RMSE	MSE	MAD	MAPE	Norm R-Sq	SSY	SSE
Lrn 100%	20	20.00	4.60000	4.60000	0.80202	1.19724	1.43339	0.91051	0.29163	0.80202	144.80000	28.66780
99%	20	20.00	4.60000	4.60000	0.80202	1.19724	1.43339	0.91051	0.29163	0.80202	144.80000	28.66780
98%	20	20.00	4.60000	4.60000	0.80202	1.19724	1.43339	0.91051	0.29163	0.80202	144.80000	28.66780
97.5%	20	20.00	4.60000	4.60000	0.80202	1.19724	1.43339	0.91051	0.29163	0.80202	144.80000	28.66780
97%	20	20.00	4.60000	4.60000	0.80202	1.19724	1.43339	0.91051	0.29163	0.80202	144.80000	28.66780
96%	20	20.00	4.60000	4.60000	0.80202	1.19724	1.43339	0.91051	0.29163	0.80202	144.80000	28.66780
95%	19	19.00	4.53325	4.68421	0.85615	1.03723	1.07586	0.80747	0.25666	0.86010	142.10526	20.44126
90%	18	18.00	4.41528	4.44444	0.87790	0.91137	0.83059	0.72215	0.25645	0.87811	122.44444	14.95070
80%	16	16.00	4.44577	4.50000	0.92814	0.73411	0.53891	0.59111	0.22806	0.92872	120.00000	8.62261
75% Q3	15	15.00	4.50636	4.46667	0.94586	0.65738	0.43214	0.53298	0.22376	0.94671	119.73333	6.48214
70%	14	14.00	4.37710	4.42857	0.96023	0.58250	0.33930	0.47705	0.22094	0.96286	119.42857	4.75025
60%	12	12.00	4.35157	4.41667	0.97708	0.42581	0.18131	0.36753	0.14923	0.97761	94.91667	2.17574
50% Median	10	10.00	4.01708	4.10000	0.98575	0.33954	0.11528	0.29808	0.15198	0.98691	80.90000	1.15284
40%	8	8.00	3.89715	4.12500	0.99096	0.28700	0.08237	0.24841	0.14806	0.99677	72.87500	0.65897
30%	6	6.00	5.00613	5.16667	0.99393	0.21765	0.04737	0.18796	0.05415	0.99771	46.83333	0.28423
25% Q1	5	5.00	5.67018	5.80000	0.99467	0.19264	0.03711	0.16272	0.03357	0.99946	34.80000	0.18555
20%	4	4.00	5.15751	5.25000	0.99626	0.16405	0.02691	0.13363	0.03324	0.99945	28.75000	0.10765
10%	2	2.00	3.04113	3.00000	0.99937	0.05003	0.00250	0.04113	0.03607	1.00000	8.00000	0.00501
5%	1	1.00	5.01266	5.00000	.	0.01266	0.00016	0.01266	0.00253	.	0.00000	0.00016
4%	1	1.00	5.01266	5.00000	.	0.01266	0.00016	0.01266	0.00253	.	0.00000	0.00016
3%	1	1.00	5.01266	5.00000	.	0.01266	0.00016	0.01266	0.00253	.	0.00000	0.00016
2.5%	1	1.00	5.01266	5.00000	.	0.01266	0.00016	0.01266	0.00253	.	0.00000	0.00016
2%	1	1.00	5.01266	5.00000	.	0.01266	0.00016	0.01266	0.00253	.	0.00000	0.00016
1%	1	1.00	5.01266	5.00000	.	0.01266	0.00016	0.01266	0.00253	.	0.00000	0.00016
95.00%	-1	19.00	4.53325	4.68421	0.85615	1.03723	1.07586	0.80747	0.25666	0.86010	142.10526	20.44126
75.00%	-5	15.00	4.50636	4.46667	0.94586	0.65738	0.43214	0.53298	0.22376	0.94671	119.73333	6.48214
50.00%	-10	10.00	4.01708	4.10000	0.98575	0.33954	0.11528	0.29808	0.15198	0.98691	80.90000	1.15284

==

Percentage of Error Statistics Due To Outliers

==

% Outliers	% MAD	Lift(MAD)	% MSE	Lift(MSE)	% MAPE	N	Wgt N

```
 Lrn      1%         15.75         15.75         28.70         28.70         19.96
1          1.00
          2%         15.75          7.88         28.70         14.35         19.96
1          1.00
        2.5%         15.75          6.30         28.70         11.48         19.96
1          1.00
          3%         15.75          5.25         28.70          9.57         19.96
1          1.00
          4%         15.75          3.94         28.70          7.17         19.96
1          1.00
          5%         15.75          3.15         28.70          5.74         19.96
1          1.00
         10%         28.62          2.86         47.85          4.78         36.36
2          2.00
         20%         48.06          2.40         69.92          3.50         55.71
4          4.00
         25% Q1      56.10          2.24         77.39          3.10         62.19
5          5.00
         30%         63.32          2.11         83.43          2.78         67.67
6          6.00
         40%         75.78          1.89         92.41          2.31         77.20
8          8.00
         50% Median  83.63          1.67         95.98          1.92         86.03
10         10.00
         60%         89.09          1.48         97.70          1.63         91.89
12         12.00
         70%         93.81          1.34         99.01          1.41         95.74
14         14.00
         75% Q3      95.53          1.27         99.35          1.32         97.12
15         15.00
         80%         97.06          1.21         99.62          1.25         98.32
16         16.00
         90%         99.55          1.11         99.98          1.11         99.50
18         18.00
         95%         99.93          1.05        100.00          1.05         99.96
19         19.00
         96%        100.00          1.04        100.00          1.04        100.00
20         20.00
         97%        100.00          1.03        100.00          1.03        100.00
20         20.00
       97.5%        100.00          1.03        100.00          1.03        100.00
20         20.00
         98%        100.00          1.02        100.00          1.02        100.00
20         20.00
         99%        100.00          1.01        100.00          1.01        100.00
20         20.00
        100%        100.00          1.00        100.00          1.00        100.00
20         20.00
-----------------------------------------------------------------------------------
       5.00%         15.75          3.15         28.70          5.74         19.96
1          1.00
      25.00%         56.10          2.24         77.39          3.10         62.19
5          5.00
      50.00%         83.63          1.67         95.98          1.92         86.03
10         10.00
-----------------------------------------------------------------------------------

 ==================================================
 Learn Sample Residual Fit Diagnostics - 3-BF Model
 ==================================================


 Grove file created: C:\Users\ayseo\AppData\Local\Temp\vejk_00482.grv: 37 kb,
 87% compression

 Grove file created containing:
      1 Mars model
```

```
 Import processed data cache : 00:00:00
 MARS model building         : 00:00:01
 Total                       : 00:00:01
>REM
```

MARS model for Y_3 in *MQ2*

```
The KEEP list has 35 variables.

 Salford Predictive Modeler(R) software suite: MARS(R) version 8.3.2.001

 Data in cache:
 N variables: 36
 N learn records: 20
                              N
 -------------------------
          Learn             20
           Test              0
        Holdout              0
 -------------------------
          Total             20

 ============
 MARS Results
 ============

 =================
 Distribution of Y
 =================

 -------------------------
 N                        20
 Sum(Weights)          20.00
 Mean                3.05000
 Median              2.50000
 Range               6.00000
 Sum                61.00000
 Cond. Mean          3.05000
 Std Dev             1.84890
 N = 0                     0
 N != 0                   20
 -------------------------
 MSE                 3.24750
 RMSE                1.80208
 MAD                 1.45000
 MAPE                0.56756
 SSY                64.95000
 SSE                64.95000
 -------------------------
      Minimum        1.00000
           1%        1.00000
           2%        1.00000
         2.5%        1.00000
           3%        1.00000
           4%        1.00000
           5%        1.00000
          10%        1.00000
          20%        1.50000
       25% Q1        2.00000
          30%        2.00000
          40%        2.00000
 -------------------------
   50% Median        2.50000
 -------------------------
          60%        3.00000
          70%        3.50000
```

```
      75% Q3       4.50000
         80%       5.00000
         90%       6.00000
         95%       6.50000
         96%       7.00000
         97%       7.00000
       97.5%       7.00000
         98%       7.00000
         99%       7.00000
     Maximum       7.00000

===============================
Forward Stepwise Knot Placement
===============================

BasFn(s)      GCV  IndBsFns EfPrms Variable                    Knot Parent    BsF
----------------------------------------------------------------------------------
    0        3.59834    0.0    1.0
    1        4.50938    1.0    5.0 X5                           1.00000
    2        5.27958    2.0    9.0 X15                          1.00000 X5     1
    3        9.34624    3.0   13.0 X27                          1.00000 X15    2
    4       23.35425    4.0   17.0 X11                          1.00000

=================================================
Final Model (After Backward Stepwise Elimination)
=================================================

Basis Fun  Coefficient Variable                     Knot  Parent
-----------------------------------------------------------------
        0      0.68510
        3      0.06119 X27                       1.00000  X15
        4      0.37018 X11                       1.00000

Piecewise Linear GCV = 2.01093, #efprms = 9.00000

========================================
ANOVA Decomposition on 2 Basis Functions
========================================

 fun    std. dev.         -gcv #bsfns  #efprms variable
-------------------------------------------------------
   1      0.81943      2.23150   1      4.00000 X11
   2      1.56820      5.29351   1      4.00000 X5
                                               X15
                                               X27
===================
Variable Importance
===================

Variable                                Importance         -gcv
---------------------------------------------------------------
X15                                      100.00000      5.29351
X5                                       100.00000      5.29351
X27                                      100.00000      5.29351
X11                                       25.92155      2.23150
X35                                        0.00000      2.01093
X16                                        0.00000      2.01093
X14                                        0.00000      2.01093
X13                                        0.00000      2.01093
X12                                        0.00000      2.01093
X10                                        0.00000      2.01093
X9                                         0.00000      2.01093
X8                                         0.00000      2.01093
X7                                         0.00000      2.01093
X6                                         0.00000      2.01093
X4                                         0.00000      2.01093
X3                                         0.00000      2.01093
X2                                         0.00000      2.01093
```

```
X17                                          0.00000      2.01093
X18                                          0.00000      2.01093
X34                                          0.00000      2.01093
X33                                          0.00000      2.01093
X32                                          0.00000      2.01093
X31                                          0.00000      2.01093
X30                                          0.00000      2.01093
X29                                          0.00000      2.01093
X28                                          0.00000      2.01093
X26                                          0.00000      2.01093
X25                                          0.00000      2.01093
X19                                          0.00000      2.01093
X20                                          0.00000      2.01093
X21                                          0.00000      2.01093
X22                                          0.00000      2.01093
X24                                          0.00000      2.01093
X23                                          0.00000      2.01093
X1                                           0.00000      2.01093

==============================
MARS Regression: Training Data
==============================

W: 20.00                                     R-SQUARED: 0.81268
MEAN DEP VAR: 3.05000                    ADJ R-SQUARED: 0.79065
                  UNCENTERED R-SQUARED = R-0 SQUARED: 0.95153

   Parameter                 Estimate         S.E.       T-Value       P-Value
-----------------------------------------------------------------------------
Constant                 |      0.68510      0.37837      1.81065      0.08791
Basis Function 3         |      0.06119      0.00752      8.13716      0.00000
Basis Function 4         |      0.37018      0.08706      4.25192      0.00054
-----------------------------------------------------------------------------
F-STATISTIC =  36.87799                    S.E. OF REGRESSION =  0.84596
    P-VALUE =  0.00000                 RESIDUAL SUM OF SQUARES =  12.16614
  [MDF,NDF] = [ 2, 17 ]              REGRESSION SUM OF SQUARES =  52.78386
-----------------------------------------------------------------------------

===============
Basis Functions
===============

BF1 = max( 0, X5 - 1);
BF2 = max( 0, X15 - 1) * BF1;
BF3 = max( 0, X27 - 1) * BF2;
BF4 = max( 0, X11 - 1);

Y = 0.685096 + 0.0611882 * BF3 + 0.370182 * BF4;

MODEL Y = BF3 BF4;

=============
Selector Info
=============

DOF Penalty = 3

BasFn  TotVar DirVar        EffPar           GCV     Learn MSE       Adj MSE    RMSE
---------------------------------------------------------------------------------
      4      4      4      17.00000      23.35426       0.52547       0.39410  0.72489
      3      4      4      13.00000       4.60962       0.56468       0.45174  0.75145
  **  2      4      4       9.00000       2.01093       0.60831       0.51706  0.77994
      1      3      3       5.00000       2.23150       1.25522       1.12970  1.12036
      0      0      0       1.00000       3.59834       3.24750          .      1.80208
---------------------------------------------------------------------------------

==============================
Regression Performance Summary
```

================================

Sample	Joint N	Wgt Joint N	Mean(Score)	Mean(Target)	R-Sq	RMSE	MSE	MAD	MAPE	Norm R-Sq	SSY	SSE
Lrn	20	20.00	3.05000	3.05000	0.81268	0.77994	0.60831	0.58466	0.27254	0.81268	64.95000	12.16614

==
Performance By Abs(Deviation) Outlier Trimming
==

Percentile	Joint N	Wgt Joint N	Mean(Score)	Mean(Target)	R-Sq	RMSE	MSE	MAD	MAPE	Norm R-Sq	SSY	SSE
Lrn 100%	20	20.00	3.05000	3.05000	0.81268	0.77994	0.60831	0.58466	0.27254	0.81268	64.95000	2.16614
99%	20	20.00	3.05000	3.05000	0.81268	0.77994	0.60831	0.58466	0.27254	0.81268	64.95000	2.16614
98%	20	20.00	3.05000	3.05000	0.81268	0.77994	0.60831	0.58466	0.27254	0.81268	64.95000	2.16614
97.5%	20	20.00	3.05000	3.05000	0.81268	0.77994	0.60831	0.58466	0.27254	0.81268	64.95000	2.16614
97%	20	20.00	3.05000	3.05000	0.81268	0.77994	0.60831	0.58466	0.27254	0.81268	64.95000	2.16614
96%	20	20.00	3.05000	3.05000	0.81268	0.77994	0.60831	0.58466	0.27254	0.81268	64.95000	2.16614
95%	19	19.00	3.03809	2.94737	0.84913	0.69567	0.48396	0.52472	0.26874	0.85175	60.94737	9.19524
90%	18	18.00	3.07669	3.05556	0.87021	0.64078	0.41060	0.47924	0.20905	0.87095	56.94444	7.39083
80%	16	16.00	3.18442	3.00000	0.92143	0.50532	0.25535	0.37850	0.17839	0.93260	52.00000	4.08559
75% Q3	15	15.00	3.11384	3.00000	0.95115	0.41151	0.16934	0.32085	0.16266	0.95756	52.00000	2.54012
70%	14	14.00	3.19991	3.14286	0.96408	0.34991	0.12244	0.27885	0.10936	0.96607	47.71429	1.71410
60%	12	12.00	3.29042	3.33333	0.98163	0.26148	0.06837	0.21585	0.07285	0.98241	44.66667	0.82048
50% Median	10	10.00	3.13688	3.10000	0.98838	0.20710	0.04289	0.17064	0.06495	0.98875	36.90000	0.42891
40%	8	8.00	2.95095	3.00000	0.99614	0.13176	0.01736	0.11814	0.05331	0.99707	36.00000	0.13888
30%	6	6.00	2.66349	2.66667	0.99729	0.10690	0.01143	0.09530	0.04936	0.99762	25.33333	0.06856
25% Q1	5	5.00	2.76303	2.80000	0.99834	0.09062	0.00821	0.08120	0.04265	0.99882	24.80000	0.04106
20%	4	4.00	2.74330	2.75000	0.99935	0.06341	0.00402	0.06198	0.04014	0.99955	24.75000	0.01608
10%	2	2.00	4.00147	4.00000	0.99968	0.05383	0.00290	0.05381	0.03138	1.00000	18.00000	0.00579
5%	1	1.00	6.94766	7.00000	.	0.05234	0.00274	0.05234	0.00748	.	0.00000	0.00274
4%	1	1.00	6.94766	7.00000	.	0.05234	0.00274	0.05234	0.00748	.	0.00000	0.00274
3%	1	1.00	6.94766	7.00000	.	0.05234	0.00274	0.05234	0.00748	.	0.00000	0.00274
2.5%	1	1.00	6.94766	7.00000	.	0.05234	0.00274	0.05234	0.00748	.	0.00000	0.00274
2%	1	1.00	6.94766	7.00000	.	0.05234	0.00274	0.05234	0.00748	.	0.00000	0.00274
1%	1	1.00	6.94766	7.00000	.	0.05234	0.00274	0.05234	0.00748	.	0.00000	0.00274
95.00%	-1	19.00	3.03809	2.94737	0.84913	0.69567	0.48396	0.52472	0.26874	0.85175	60.94737	9.19524

75.00%	-5	15.00	3.11384	3.00000	0.95115	0.41151	0.16934	0.32085	0.16266	0.95756	52.00000	2.54012
50.00%	-10	10.00	3.13688	3.10000	0.98838	0.20710	0.04289	0.17064	0.06495	0.98875	36.90000	0.42891

==
Percentage of Error Statistics Due To Outliers
==

% Outliers	% MAD	Lift(MAD)	% MSE	Lift(MSE)	% MAPE	N	Wgt N
Lrn 1%	14.74	14.74	24.42	24.42	24.64	1	1.00
2%	14.74	7.37	24.42	12.21	24.64	1	1.00
2.5%	14.74	5.90	24.42	9.77	24.64	1	1.00
3%	14.74	4.91	24.42	8.14	24.64	1	1.00
4%	14.74	3.69	24.42	6.10	24.64	1	1.00
5%	14.74	2.95	24.42	4.88	24.64	1	1.00
10%	26.23	2.62	39.25	3.93	41.32	2	2.00
20%	48.21	2.41	66.42	3.32	60.98	4	4.00
25% Q1	58.84	2.35	79.12	3.16	68.14	5	5.00
30%	66.61	2.22	85.91	2.86	74.47	6	6.00
40%	77.85	1.95	93.26	2.33	83.96	8	8.00
50% Median	85.41	1.71	96.47	1.93	89.20	10	10.00
60%	91.92	1.53	98.86	1.65	92.77	12	12.00
70%	95.11	1.36	99.44	1.42	95.57	14	14.00
75% Q3	96.53	1.29	99.66	1.33	96.59	15	15.00
80%	97.88	1.22	99.87	1.25	97.60	16	16.00
90%	99.08	1.10	99.95	1.11	99.35	18	18.00
95%	99.55	1.05	99.98	1.05	99.86	19	19.00
96%	100.00	1.04	100.00	1.04	100.00	20	20.00
97%	100.00	1.03	100.00	1.03	100.00	20	20.00
97.5%	100.00	1.03	100.00	1.03	100.00	20	20.00
98%	100.00	1.02	100.00	1.02	100.00	20	20.00
99%	100.00	1.01	100.00	1.01	100.00	20	20.00
100%	100.00	1.00	100.00	1.00	100.00	20	20.00
5.00%	14.74	2.95	24.42	4.88	24.64	1	1.00
25.00%	58.84	2.35	79.12	3.16	68.14	5	5.00
50.00%	85.41	1.71	96.47	1.93	89.20	10	10.00

```
 =====================================================
 Learn Sample Residual Fit Diagnostics - 2-BF Model
 =====================================================


 Grove file created: C:\Users\ayseo\AppData\Local\Temp\vejk_00673.grv: 31 kb,
 90% compression

 Grove file created containing:
      1 Mars model

 Import processed data cache : 00:00:00
 MARS model building         : 00:00:02
 Total                       : 00:00:02
>REM
>
```

MARS model for Y_4 in *MQ2*

```
The KEEP list has 35 variables.

 Salford Predictive Modeler(R) software suite: MARS(R) version 8.3.2.001

 Data in cache:
 N variables: 36
 N learn records: 20
                         N
 -------------------------
          Learn         20
           Test          0
        Holdout          0
 -------------------------
          Total         20

 ============
 MARS Results
 ============

 =================
 Distribution of Y
 =================

 -------------------------
 N                      20
 Sum(Weights)        20.00
 Mean              4.75000
 Median            4.50000
 Range             7.00000
 Sum              95.00000
 Cond. Mean        4.75000
 Std Dev           2.48945
 N = 0                   0
 N != 0                 20
 -------------------------
 MSE               5.88750
 RMSE              2.42642
 MAD               2.15000
 MAPE              0.84696
 SSY             117.75000
 SSE             117.75000
 -------------------------
      Minimum      1.00000
           1%      1.00000
           2%      1.00000
         2.5%      1.00000
```

```
          3%       1.00000
          4%       1.00000
          5%       1.00000
         10%       1.00000
         20%       2.50000
      25% Q1       3.00000
         30%       3.00000
         40%       4.00000
-------------------------
  50% Median       4.50000
-------------------------
         60%       6.00000
         70%       6.50000
      75% Q3       7.00000
         80%       7.50000
         90%       8.00000
         95%       8.00000
         96%       8.00000
         97%       8.00000
       97.5%       8.00000
         98%       8.00000
         99%       8.00000
     Maximum       8.00000

===============================
Forward Stepwise Knot Placement
===============================

BasFn(s)      GCV  IndBsFns EfPrms Variable                    Knot   Parent   BsF
-----------------------------------------------------------------------------------
    0        6.52355    0.0    1.0
    1        5.60196    1.0    5.0 X24                         1.00000
    2        7.01640    2.0    9.0 X8                          1.00000
    3       12.80679    3.0   13.0 X5                          1.00000 X8      2
    4       44.33646    4.0   17.0 X30                         1.00000 X24     1

=================================================
Final Model (After Backward Stepwise Elimination)
=================================================

Basis Fun  Coefficient Variable                      Knot   Parent
---------------------------------------------------------------
        0      7.50754
        3     -0.16046 X5                         1.00000   X8
        4     -0.16180 X30                        1.00000   X24

Piecewise Linear GCV = 3.76956, #efprms = 9.00000

========================================
ANOVA Decomposition on 2 Basis Functions
========================================

 fun     std. dev.         -gcv #bsfns  #efprms variable
-------------------------------------------------------
   1      1.25416      4.82011   1      4.00000 X5
                                                X8
   2      1.73858      7.39431   1      4.00000 X24
                                                X30

===================
Variable Importance
===================

Variable                                Importance         -gcv
---------------------------------------------------------------
X30                                      100.00000      7.39431
X24                                      100.00000      7.39431
X5                                        53.83561      4.82011
```

```
X8                                        53.83561      4.82011
X35                                        0.00000      3.76956
X15                                        0.00000      3.76956
X14                                        0.00000      3.76956
X13                                        0.00000      3.76956
X12                                        0.00000      3.76956
X11                                        0.00000      3.76956
X10                                        0.00000      3.76956
X9                                         0.00000      3.76956
X7                                         0.00000      3.76956
X6                                         0.00000      3.76956
X4                                         0.00000      3.76956
X3                                         0.00000      3.76956
X2                                         0.00000      3.76956
X16                                        0.00000      3.76956
X17                                        0.00000      3.76956
X18                                        0.00000      3.76956
X34                                        0.00000      3.76956
X33                                        0.00000      3.76956
X32                                        0.00000      3.76956
X31                                        0.00000      3.76956
X29                                        0.00000      3.76956
X28                                        0.00000      3.76956
X27                                        0.00000      3.76956
X26                                        0.00000      3.76956
X19                                        0.00000      3.76956
X20                                        0.00000      3.76956
X21                                        0.00000      3.76956
X22                                        0.00000      3.76956
X25                                        0.00000      3.76956
X23                                        0.00000      3.76956
X1                                         0.00000      3.76956

==============================
MARS Regression: Training Data
==============================

W: 20.00                                   R-SQUARED: 0.80632
MEAN DEP VAR: 4.75000                  ADJ R-SQUARED: 0.78353
                  UNCENTERED R-SQUARED = R-0 SQUARED: 0.95992

   Parameter                   Estimate         S.E.       T-Value       P-Value
 -----------------------------------------------------------------------------
Constant                |       7.50754      0.41954      17.89460       0.00000
Basis Function 3        |      -0.16046      0.03316      -4.83958       0.00015
Basis Function 4        |      -0.16180      0.02412      -6.70886       0.00000
 -----------------------------------------------------------------------------
F-STATISTIC =   35.38678                    S.E. OF REGRESSION =   1.15824
    P-VALUE =   0.00000                RESIDUAL SUM OF SQUARES =   22.80584
  [MDF,NDF] = [ 2, 17 ]              REGRESSION SUM OF SQUARES =   94.94416
 -----------------------------------------------------------------------------

===============
Basis Functions
===============

BF1 = max( 0, X24 - 1);
BF2 = max( 0, X8 - 1);
BF3 = max( 0, X5 - 1) * BF2;
BF4 = max( 0, X30 - 1) * BF1;

Y = 7.50754 - 0.160461 * BF3 - 0.1618 * BF4;

MODEL Y = BF3 BF4;

=============
Selector Info
=============
```

DOF Penalty = 3

	BasFn	TotVar	DirVar	EffPar	GCV	Learn MSE	Adj MSE	RMSE
	4	4	4	17.00000	44.33645	0.99757	0.74818	0.99878
	3	4	4	13.00000	8.22375	1.00741	0.80593	1.00370
**	2	4	4	9.00000	3.76956	1.14029	0.96925	1.06784
	1	2	2	5.00000	4.82011	2.71131	2.44018	1.64661
	0	0	0	1.00000	6.52355	5.88750	.	2.42642

================================
Regression Performance Summary
================================

Sample	Joint N	Wgt Joint N	Mean(Score)	Mean(Target)	R-Sq	RMSE	MSE	MAD	MAPE	Norm R-Sq	SSY	SSE
Lrn	20	20.00	4.75000	4.75000	0.80632	1.06784	1.14029	0.82419	0.35382	0.80632	117.75000	22.80584

===
Performance By Abs(Deviation) Outlier Trimming
===

Percentile	Joint N	Wgt Joint N	Mean(Score)	Mean(Target)	R-Sq	RMSE	MSE	MAD	MAPE	Norm R-Sq	SSY	SSE
Lrn 100%	20	20.00	4.75000	4.75000	0.80632	1.06784	1.14029	0.82419	0.35382	0.80632	117.75000	22.80584
99%	20	20.00	4.75000	4.75000	0.80632	1.06784	1.14029	0.82419	0.35382	0.80632	117.75000	22.80584
98%	20	20.00	4.75000	4.75000	0.80632	1.06784	1.14029	0.82419	0.35382	0.80632	117.75000	22.80584
97.5%	20	20.00	4.75000	4.75000	0.80632	1.06784	1.14029	0.82419	0.35382	0.80632	117.75000	22.80584
97%	20	20.00	4.75000	4.75000	0.80632	1.06784	1.14029	0.82419	0.35382	0.80632	117.75000	22.80584
96%	20	20.00	4.75000	4.75000	0.80632	1.06784	1.14029	0.82419	0.35382	0.80632	117.75000	22.80584
95%	19	19.00	4.80052	4.94737	0.85409	0.88914	0.79057	0.72071	0.22559	0.85889	102.94737	15.02088
90%	18	18.00	4.76691	5.05556	0.90667	0.71626	0.51303	0.62712	0.19358	0.92198	98.94444	9.23456
80%	16	16.00	4.82741	5.00000	0.92722	0.62546	0.39120	0.55337	0.18187	0.93277	86.00000	6.25917
75% Q3	15	15.00	4.94915	5.06667	0.93805	0.59226	0.35078	0.52368	0.17735	0.94101	84.93333	5.26166
70%	14	14.00	5.15897	5.21429	0.94668	0.55323	0.30606	0.49049	0.16648	0.95112	80.35714	4.28491
60%	12	12.00	5.18285	5.25000	0.95340	0.49166	0.24173	0.43361	0.15042	0.95453	62.25000	2.90073
50% Median	10	10.00	5.42648	5.50000	0.95584	0.42292	0.17886	0.37136	0.09841	0.96069	40.50000	1.78865
40%	8	8.00	5.04662	5.12500	0.96759	0.36496	0.13320	0.31449	0.10146	0.97590	32.87500	1.06556
30%	6	6.00	4.90048	5.00000	0.98146	0.29414	0.08652	0.24516	0.10080	0.98422	28.00000	0.51910
25% Q1	5	5.00	4.37907	4.40000	0.98392	0.23519	0.05532	0.19570	0.10864	0.99463	17.20000	0.27658
20%	4	4.00	5.33362	5.25000	0.96955	0.14468	0.02093	0.13483	0.02602	0.98007	2.75000	0.08373
10%	2	2.00	5.48906	5.50000	0.96605	0.09213	0.00849	0.09148	0.01659	1.00000	0.50000	0.01698
5%	1	1.00	5.08054	5.00000	.	0.08054	0.00649	0.08054	0.01611	.	0.00000	0.00649

```
         4%          1          1.00       5.08054      5.00000          .
0.08054      0.00649      0.08054      0.01611          .            0.00000    0.00649
         3%          1          1.00       5.08054      5.00000          .
0.08054      0.00649      0.08054      0.01611          .            0.00000    0.00649
       2.5%          1          1.00       5.08054      5.00000          .
0.08054      0.00649      0.08054      0.01611          .            0.00000    0.00649
         2%          1          1.00       5.08054      5.00000          .
0.08054      0.00649      0.08054      0.01611          .            0.00000    0.00649
         1%          1          1.00       5.08054      5.00000          .
0.08054      0.00649      0.08054      0.01611          .            0.00000     0.00649
-----------------------------------------------------------------------------------
     95.00%         -1         19.00       4.80052      4.94737      0.85409
0.88914      0.79057      0.72071      0.22559      0.85889    102.94737    15.02088
     75.00%         -5         15.00       4.94915      5.06667      0.93805
0.59226      0.35078      0.52368      0.17735      0.94101     84.93333     5.26166
     50.00%        -10         10.00       5.42648      5.50000      0.95584
0.42292      0.17886      0.37136      0.09841      0.96069     40.50000     1.78865

 ==============================================
 Percentage of Error Statistics Due To Outliers
 ==============================================

 % Outliers         % MAD    Lift(MAD)         % MSE    Lift(MSE)         % MAPE
N         Wgt N
-----------------------------------------------------------------------------------
 Lrn     1%         16.93        16.93         34.14        34.14          39.43
1          1.00
         2%         16.93         8.46         34.14        17.07          39.43
1          1.00
       2.5%         16.93         6.77         34.14        13.65          39.43
1          1.00
         3%         16.93         5.64         34.14        11.38          39.43
1          1.00
         4%         16.93         4.23         34.14         8.53          39.43
1          1.00
         5%         16.93         3.39         34.14         6.83          39.43
1          1.00
        10%         31.52         3.15         59.51         5.95          50.76
2          2.00
        20%         46.29         2.31         72.55         3.63          66.99
4          4.00
        25% Q1      52.35         2.09         76.93         3.08          73.10
5          5.00
        30%         58.34         1.94         81.21         2.71          79.09
6          6.00
        40%         68.43         1.71         87.28         2.18          87.27
8          8.00
        50% Median  77.47         1.55         92.16         1.84          91.18
10         10.00
        60%         84.74         1.41         95.33         1.59          94.20
12         12.00
        70%         91.08         1.30         97.72         1.40          96.63
14         14.00
        75% Q3      94.06         1.25         98.79         1.32          97.66
15         15.00
        80%         96.73         1.21         99.63         1.25          98.53
16         16.00
        90%         98.89         1.10         99.93         1.11          99.53
18         18.00
        95%         99.51         1.05         99.97         1.05          99.77
19         19.00
        96%        100.00         1.04        100.00         1.04         100.00
20         20.00
        97%        100.00         1.03        100.00         1.03         100.00
20         20.00
      97.5%        100.00         1.03        100.00         1.03         100.00
20         20.00
```

```
       98%      100.00         1.02       100.00         1.02       100.00
20        20.00
       99%      100.00         1.01       100.00         1.01       100.00
20        20.00
      100%      100.00         1.00       100.00         1.00       100.00
20        20.00
--------------------------------------------------------------------------------
     5.00%       16.93         3.39        34.14         6.83        39.43
1         1.00
    25.00%       52.35         2.09        76.93         3.08        73.10
5         5.00
    50.00%       77.47         1.55        92.16         1.84        91.18
10        10.00
--------------------------------------------------------------------------------

 ==================================================
 Learn Sample Residual Fit Diagnostics - 2-BF Model
 ==================================================


 Grove file created: C:\Users\ayseo\AppData\Local\Temp\vejk_00963.grv: 38 kb,
 91% compression

 Grove file created containing:
      1 Mars model

 Import processed data cache : 00:00:00
 MARS model building         : 00:00:02
 Total                       : 00:00:02
>REM
```

MARS model for Y_5 in *MQ2*

```
The KEEP list has 35 variables.

 Salford Predictive Modeler(R) software suite: MARS(R) version 8.3.2.001

 Data in cache:
 N variables: 36
 N learn records: 20

                         N
 -------------------------
          Learn         20
           Test          0
        Holdout          0
 -------------------------
          Total         20

 ============
 MARS Results
 ============

 =================
 Distribution of Y
 =================

 -------------------------
 N                      20
 Sum(Weights)        20.00
 Mean              3.55000
 Median            3.50000
 Range             5.00000
 Sum              71.00000
 Cond. Mean        3.55000
 Std Dev           1.76143
```

```
N = 0                      0
N != 0                    20
-------------------------
MSE                  2.94750
RMSE                 1.71683
MAD                  1.55000
MAPE                 0.70000
SSY                 58.95000
SSE                 58.95000
-------------------------
     Minimum         1.00000
          1%         1.00000
          2%         1.00000
        2.5%         1.00000
          3%         1.00000
          4%         1.00000
          5%         1.00000
         10%         1.00000
         20%         2.00000
      25% Q1         2.00000
         30%         2.00000
         40%         3.00000
-------------------------
  50% Median         3.50000
-------------------------
         60%         4.50000
         70%         5.00000
      75% Q3         5.00000
         80%         5.00000
         90%         6.00000
         95%         6.00000
         96%         6.00000
         97%         6.00000
       97.5%         6.00000
         98%         6.00000
         99%         6.00000
     Maximum         6.00000

===============================
Forward Stepwise Knot Placement
===============================

BasFn(s)      GCV   IndBsFns EfPrms Variable                   Knot Parent    BsF
---------------------------------------------------------------------------------
    0        3.26593    0.0     1.0
    1        2.72517    1.0     5.0 X21                       2.00000
    2        2.59770    2.0     9.0 X9                        1.00000 X21     1
    3        4.27871    3.0    13.0 X13                       1.00000
    4       16.60963    4.0    17.0 X10                       1.00000 X13     3


=================================================
Final Model (After Backward Stepwise Elimination)
=================================================

Basis Fun  Coefficient Variable                  Knot   Parent
-----------------------------------------------------------------
        0      4.68787
        1     -1.00507 X21                       2.00000
        2      0.16549 X9                        1.00000 X21

Piecewise Linear GCV = 2.59770, #efprms = 9.00000


========================================
ANOVA Decomposition on 2 Basis Functions
========================================
```

```
 fun     std. dev.         -gcv #bsfns  #efprms variable
-------------------------------------------------------
   1      2.04502      5.15201   1      4.00000 X21
   2      1.21625      2.72517   1      4.00000 X9
                                                X21
```

```
===================
Variable Importance
===================
```

Variable	Importance	-gcv
X21	100.00000	3.26593
X9	43.67618	2.72517
X35	0.00000	2.59770
X16	0.00000	2.59770
X15	0.00000	2.59770
X14	0.00000	2.59770
X13	0.00000	2.59770
X12	0.00000	2.59770
X11	0.00000	2.59770
X10	0.00000	2.59770
X8	0.00000	2.59770
X7	0.00000	2.59770
X6	0.00000	2.59770
X5	0.00000	2.59770
X4	0.00000	2.59770
X3	0.00000	2.59770
X2	0.00000	2.59770
X17	0.00000	2.59770
X18	0.00000	2.59770
X34	0.00000	2.59770
X33	0.00000	2.59770
X32	0.00000	2.59770
X31	0.00000	2.59770
X30	0.00000	2.59770
X29	0.00000	2.59770
X28	0.00000	2.59770
X27	0.00000	2.59770
X26	0.00000	2.59770
X19	0.00000	2.59770
X20	0.00000	2.59770
X22	0.00000	2.59770
X23	0.00000	2.59770
X25	0.00000	2.59770
X24	0.00000	2.59770
X1	0.00000	2.59770

```
==============================
MARS Regression: Training Data
==============================

W: 20.00                                 R-SQUARED: 0.73340
MEAN DEP VAR: 3.55000                ADJ R-SQUARED: 0.70203
                  UNCENTERED R-SQUARED = R-0 SQUARED: 0.94947
```

Parameter	Estimate	S.E.	T-Value	P-Value
Constant	4.68787	0.33894	13.83091	0.00000
Basis Function 1	-1.00507	0.14868	-6.75981	0.00000
Basis Function 2	0.16549	0.04116	4.02030	0.00089

```
F-STATISTIC =  23.38290                 S.E. OF REGRESSION =  0.96150
    P-VALUE =  0.00001              RESIDUAL SUM OF SQUARES =  15.71610
  [MDF,NDF] = [ 2, 17 ]           REGRESSION SUM OF SQUARES =  43.23390
-----------------------------------------------------------------------------

===============
```

```
Basis Functions
===============

BF1 = max( 0, X21 - 2);
BF2 = max( 0, X9 - 1) * BF1;

Y = 4.68787 - 1.00507 * BF1 + 0.165495 * BF2;

MODEL Y = BF1 BF2;

=============
Selector Info
=============

DOF Penalty = 3

BasFn  TotVar DirVar      EffPar          GCV     Learn MSE       Adj MSE     RMSE
-------------------------------------------------------------------------------------
     4      4      4    17.00000     16.60963       0.37372       0.28029  0.61132
     3      3      3    13.00000      4.27871       0.52414       0.41931  0.72398
 **  2      2      2     9.00000      2.59770       0.78581       0.66793  0.88646
     1      1      1     5.00000      2.72517       1.53291       1.37962  1.23811
     0      0      0     1.00000      3.26593       2.94750             .  1.71683
-------------------------------------------------------------------------------------

==============================
Regression Performance Summary
==============================

Sample       Joint N  Wgt Joint N  Mean(Score) Mean(Target)        R-Sq
RMSE           MSE           MAD         MAPE     Norm R-Sq           SSY           SSE
-------------------------------------------------------------------------------------
Lrn               20        20.00      3.55000      3.55000      0.73340
0.88646      0.78581      0.73021      0.28964      0.73340     58.95000       5.71611


==============================================
Performance By Abs(Deviation) Outlier Trimming
==============================================

Percentile   Joint N  Wgt Joint N  Mean(Score) Mean(Target)        R-Sq
RMSE           MSE           MAD         MAPE     Norm R-Sq           SSY           SSE
-------------------------------------------------------------------------------------
Lrn    100%       20        20.00      3.55000      3.55000      0.73340
0.88646      0.78581      0.73021      0.28964      0.73340     58.95000      15.71611
        99%       20        20.00      3.55000      3.55000      0.73340
0.88646      0.78581      0.73021      0.28964      0.73340     58.95000      15.71611
        98%       20        20.00      3.55000      3.55000      0.73340
0.88646      0.78581      0.73021      0.28964      0.73340     58.95000      15.71611
      97.5%       20        20.00      3.55000      3.55000      0.73340
0.88646      0.78581      0.73021      0.28964      0.73340     58.95000      15.71611
        97%       20        20.00      3.55000      3.55000      0.73340
0.88646      0.78581      0.73021      0.28964      0.73340     58.95000      15.71611
        96%       20        20.00      3.55000      3.55000      0.73340
0.88646      0.78581      0.73021      0.28964      0.73340     58.95000      15.71611
        95%       19        19.00      3.52559      3.63158      0.79333
0.78341      0.61373      0.66265      0.25189      0.79751     56.42105      11.66079
        90%       18        18.00      3.74088      3.77778      0.79965
0.73934      0.54663      0.62449      0.19091      0.82778     49.11111       9.83935
        80%       16        16.00      3.62251      3.50000      0.83169
0.63226      0.39975      0.53853      0.18743      0.84977     38.00000       6.39596
        75% Q3    15        15.00      3.65320      3.60000      0.85827
0.57997      0.33637      0.49696      0.16119      0.86858     35.60000       5.04549
        70%       14        14.00      3.62744      3.64286      0.88591
0.53571      0.28698      0.46005      0.14857      0.89784     35.21429       4.01773
        60%       12        12.00      3.49511      3.66667      0.93340
0.43864      0.19240      0.38315      0.12904      0.97586     34.66667       2.30882
```

```
       50% Median 10        10.00       3.23072      3.30000      0.94730
0.37087      0.13755      0.32318      0.12986      0.97371     26.10000    1.37548
        40%          8         8.00       3.14293      3.25000      0.96704
0.29761      0.08857      0.26107      0.10924      0.98906     21.50000    0.70857
        30%          6         6.00       2.99660      3.00000      0.98063
0.24106      0.05811      0.20873      0.11008      0.99636     18.00000    0.34867
        25% Q1       5         5.00       3.33001      3.40000      0.98181
0.21912      0.04801      0.18456      0.06618      0.99608     13.20000    0.24007
        20%          4         4.00       2.99358      3.00000      0.98651
0.18367      0.03373      0.14964      0.06652      0.99598     10.00000    0.13494
        10%          2         2.00       2.57076      2.50000      0.99633
0.09085      0.00825      0.07076      0.06559      1.00000      4.50000    0.01651
         5%          1         1.00       4.01378      4.00000       .
0.01378      0.00019      0.01378      0.00345       .            0.00000    0.00019
         4%          1         1.00       4.01378      4.00000       .
0.01378      0.00019      0.01378      0.00345       .            0.00000    0.00019
         3%          1         1.00       4.01378      4.00000       .
0.01378      0.00019      0.01378      0.00345       .            0.00000    0.00019
       2.5%          1         1.00       4.01378      4.00000       .
0.01378      0.00019      0.01378      0.00345       .            0.00000    0.00019
         2%          1         1.00       4.01378      4.00000       .
0.01378      0.00019      0.01378      0.00345       .            0.00000    0.00019
         1%          1         1.00       4.01378      4.00000       .
0.01378      0.00019      0.01378      0.00345       .            0.00000    0.00019
-----------------------------------------------------------------------------------------
      95.00%        -1        19.00       3.52559      3.63158      0.79333
0.78341      0.61373      0.66265      0.25189      0.79751     56.42105   11.66079
      75.00%        -5        15.00       3.65320      3.60000      0.85827
0.57997      0.33637      0.49696      0.16119      0.86858     35.60000    5.04549
      50.00%       -10        10.00       3.23072      3.30000      0.94730
0.37087      0.13755      0.32318      0.12986      0.97371     26.10000    1.37548

 ==================================================
 Percentage of Error Statistics Due To Outliers
 ==================================================

 % Outliers          % MAD     Lift(MAD)         % MSE     Lift(MSE)         % MAPE
N         Wgt N
-----------------------------------------------------------------------------------------
 Lrn      1%         13.79        13.79         25.80        25.80         23.30
1          1.00
          2%         13.79         6.89         25.80        12.90         23.30
1          1.00
        2.5%         13.79         5.52         25.80        10.32         23.30
1          1.00
          3%         13.79         4.60         25.80         8.60         23.30
1          1.00
          4%         13.79         3.45         25.80         6.45         23.30
1          1.00
          5%         13.79         2.76         25.80         5.16         23.30
1          1.00
         10%         23.03         2.30         37.39         3.74         40.68
2          2.00
         20%         41.00         2.05         59.30         2.97         56.54
4          4.00
         25% Q1      48.96         1.96         67.90         2.72         62.23
5          5.00
         30%         55.90         1.86         74.44         2.48         67.87
6          6.00
         40%         68.52         1.71         85.31         2.13         77.05
8          8.00
         50% Median  77.87         1.56         91.25         1.82         84.60
10         10.00
         60%         85.70         1.43         95.49         1.59         89.11
12         12.00
         70%         91.42         1.31         97.78         1.40         93.11
14         14.00
```

```
        75% Q3      93.68          1.25         98.47          1.31         94.80
15        15.00
        80%         95.90          1.20         99.14          1.24         96.49
16        16.00
        90%         99.03          1.10         99.89          1.11         98.86
18        18.00
        95%         99.91          1.05        100.00          1.05         99.94
19        19.00
        96%        100.00          1.04        100.00          1.04        100.00
20        20.00
        97%        100.00          1.03        100.00          1.03        100.00
20        20.00
        97.5%      100.00          1.03        100.00          1.03        100.00
20        20.00
        98%        100.00          1.02        100.00          1.02        100.00
20        20.00
        99%        100.00          1.01        100.00          1.01        100.00
20        20.00
       100%        100.00          1.00        100.00          1.00        100.00
20        20.00
-----------------------------------------------------------------------------------
      5.00%         13.79          2.76         25.80          5.16         23.30
1         1.00
     25.00%         48.96          1.96         67.90          2.72         62.23
5         5.00
     50.00%         77.87          1.56         91.25          1.82         84.60
10        10.00
-----------------------------------------------------------------------------------

 ==================================================
 Learn Sample Residual Fit Diagnostics - 2-BF Model
 ==================================================

 Grove file created: C:\Users\ayseo\AppData\Local\Temp\vcu0_00057.grv: 28 kb,
 82% compression

 Grove file created containing:
      1 Mars model

 Import processed data cache : 00:00:00
 MARS model building         : 00:00:00
 Total                       : 00:00:00
>REM
```

Appendix E. Formulas for *m* and *TRC* Matrices, *x*'s and *xx*'s

https://docs.google.com/spreadsheets/d/1eyFprG2wFZVWyFhcGDES0wJ2cqp9wu6q/edit?usp=sharing&ouid=104362461797002450951&rtpof=true&sd=true

Appendix F. GAMS Results

https://drive.google.com/drive/folders/1atUI-4DtJGft53OZrsld8s1esc95ZimS?usp=drive_link

Appendix G. *MQ*1&*MQ*2, *R* Data

https://drive.google.com/drive/folders/1nrggTXVU0XW5x1v1t--zNi5ZsrW5vgxA?usp=drive_link

References

Agrawal, O. P. (2010). Generalized variational problems and Euler–Lagrange equations. *Computers & Mathematics with Applications*, *59*(5), 1852-1864.

Ahram, T., Karwowski, W., & Taiar, R. (2022). A framework for human-centered production planning and control in smart manufacturing. International Journal of Advanced Manufacturing Technology, *65*(4), 1012-1025. Retrieved from https://www.researchgate.net/publication/363814482

Almeida, M. T., & Duarte, B. P. (2011). Multi-period stochastic production planning with demand uncertainty. International Journal of Production Economics, 131(1), 73-82. https://doi.org/10.1016/j.ijpe.2010.12.012

Anand Jayakumar, A., Krishnaraj, C., & Nachimuthu, A. K. (2017). *Aggregate production planning: Mixed strategy*. Pakistan Journal of Biotechnology, 14(3), 487-490.

Aouni, B., Colapinto, C., & La Torre, D. (2014). Financial portfolio management through the goal programming model: Current state-of-the-art. European Journal of Operational Research, 234(2), 536-545.

Aster, R. C., Borchers B. and Thurber C. H. (2018). Parameter Estimation and Inverse Problems. Elsevier, Paperback. Third Edition. ISBN: 9780128046517.

Attia, E. A., Megahed, A., AlArjani, A., Elbetar, A., & Duquenne, P. (2022). Aggregate production planning considering organizational learning with case based analysis. Ain Shams Engineering Journal, 13(2), 101575.

Aydin, N. S., & Tirkolaee, E. B. (2022). A systematic review of aggregate production planning literature with an outlook for sustainability and circularity. Environment, development and sustainability, 1-42.

Balakrishnan, J., & Cheng, C. H. (2007). The theory and practice of DSS in aggregate production planning. Computers & Industrial Engineering, *53*(2), 275-289.

Bechtold, S. E., & Jacobs, L. W. (1990). Implicit modeling of flexible work shifts. *Management Science, 36*(11), 1339–1351.

Bellman, R. (1952). On the theory of dynamic programming. *Proceedings of the national Academy of Sciences*, *38*(8), 716-719.

Bellman, R. (1954). The theory of dynamic programming. *Bulletin of the American Mathematical Society*, *60*(6), 503-515.

Bellman, R., & Kalaba, R. (1957). Dynamic programming and statistical communication theory. *Proceedings of the National Academy of Sciences*, *43*(8), 749-751.

Bellman, R. E., & Zadeh, L. A. (1970). Decision-making in a fuzzy environment. Management science, 17(4), B-141.

Ben-Tal, A., & Nemirovski, A. (2002). Robust optimization–methodology anD applications. *Mathematical programming*, *92*, 453-480.

Beratan, K. K. (2007). A cognition-based view of decision processes in complex social–ecological systems. Ecology and Society, 12(1). https://www.jstor.org/stable/26267857

Bertsimas, D., & Sim, M. (2003). Robust discrete optimization and network flows. *Mathematical programming*, *98*(1), 49-71.

Bertsimas, D., & Thiele, A. (2006). A Robust Optimization Approach to Inventory Theory. *Operations Research*, 54(1), 150-168.

Birge, J. R., & Louveaux, F. (2001). *Introduction to stochastic programming* (2nd ed.). Springer. https://doi.org/10.1007/978-1-4614-0237-4

Bitran, G. R., & Tirupati, D. (1993). Hierarchical production planning. *Handbooks in* operations *research and management science*, *4*, 523-568.

Bonekamp, L., & Sure, M. (2015). Consequences of Industry 4.0 on human labor and work organization. *Journal of Business and Industrial Engineering, 8*(2), 35-40.

Bowman, E. H. (1963). Consistency and optimality in managerial decision-making. *Management Science, 9*(2), 310–321. https://doi.org/10.1287/mnsc.9.2.310

Brough, P., Johnson, G., Drummond, S., Pennisi, S., & Timms, C. (2011). Comparisons of cognitive ability and job attitudes of older and younger workers. Equality, Diversity and Inclusion: An International Journal, 30(2), 105–126. https://doi.org/10.1108/02610151111116508

Buffa, E. S., & Sarin, R. K. (1987). Modern production/operations management. *Wiley*.

Buxey, G. M. (1979). Production planning under uncertainties: A review and some extensions. Omega, 7(6), 457-465. https://doi.org/10.1016/0305-0483(79)90049-1

Carayannis, E. G., & Morawska-Jancelewicz, K. (2022). Industry 5.0: Towards a human-centric and sustainable paradigm shift**.** *Journal of the Knowledge Economy, 13*(1), 44-59. https://doi.org/10.1007/s13132-021-00811-3

Charnes, A., Cooper, W. W., & Ferguson, R. O. (1955). Optimal estimation of executive compensation by linear programming. Management science, 1(2), 138-151.

Charnes, A., & Cooper, W. W. (1957). Management models and industrial applications of linear programming. Management science, 4(1), 38-91.

Charnes, A., & Cooper, W. W. (1977). Goal programming and multiple objective optimizations: Part 1. European journal of operational research, 1(1), 39-54.

Chaturvedi, N. D. (2017). Minimizing energy consumption via multiple installations aggregate production planning. Clean Technologies and Environmental Policy, 19(7), 1977-1984.

Chen, G., & Pham, T. T. (2000). Introduction to fuzzy sets, fuzzy logic, and fuzzy control systems. CRC press.

Chen, Y., Zhou, Y., & Zhang, Y. (2020, November). Collaborative Production Planning with Unknown Parameters using Model Predictive Control and Machine Learning. In 2020 Chinese Automation Congress (CAC) (pp. 2185-2190). IEEE.

Cheraghalikhani, A., Khoshalhan, F., & Mokhtari, H. (2019). Aggregate production planning: A literature review and future research directions. International Journal of Industrial Engineering Computations, 10(2), 309-330.

Chopra, S., & Meindl, P. (2021). *Supply Chain Management: Strategy, Planning, and Operation* (8th ed.). Pearson.

Cristofaro, M. (2020). “I feel and think, therefore I am”: An Affect-Cognitive Theory of management decisions. European Management Journal, 38(2), 344–355.

Dantzig, G. B. (1947). *Maximization of a Linear Function of Variables Subject to Linear Inequalities*. In T. C. Koopmans (Ed.), *Activity Analysis of Production and Allocation* (pp. 339-347). Wiley.

Darvishi, F., Yaghin, R. G., & Sadeghi, A. (2020). Integrated fabric procurement and multi-site apparel production planning with cross-docking: a hybrid fuzzy-robust stochastic programming approach. Applied Soft Computing, 92, 106267.

Data Mining, Machine Learning & Predictive Analytics Software | Minitab. (n.d.). Retrieved 2 November 2024, from https://www.minitab.com/en-us/products/spm/

Davis, M. H. (1984). Piecewise-deterministic Markov processes: A general class of non-diffusion stochastic models. *Journal of the Royal Statistical Society: Series B (Methodological)*, *46*(3), 353-376.

Demartini, M. (2022). Human-centric design and sustainability in Industry 5.0. *Sustainability, 14*(2), 987-1003. https://doi.org/10.3390/su14020987

Djordjevic, I., Petrovic, D., & Stojic, G. (2019). A fuzzy linear programming model for aggregated production planning (APP) in the automotive industry. Computers in Industry, 110, 48-63.

Dohale, V., Ambilkar, P., Gunasekaran, A., & Bilolikar, V. (2022). A multi-product and multi-period aggregate production plan: a case of automobile component manufacturing firm. Benchmarking: An International Journal, 29(10), 3396-3425.

Dzielinski, B. P., & Gomory, R. E. (1965). Optimal programming of lot sizes, inventory and labor allocations. *Management Science*, *11*(9), 874-890.

Eilon, S. (1975). Five approaches to aggregate production planning. AIIE Transactions, 7(2), 118-131.

Entezaminia, A., Heidari, M., & Rahmani, D. (2017). Robust aggregate production planning in a green supply chain under uncertainty considering reverse logistics: a case study. The International Journal of Advanced Manufacturing Technology, 90(5-8), 1507-1528.

Fahimnia, B., Tang, C. S., Davarzani, H., & Sarkis, J. (2015). Quantitative models for managing supply chain risks: A review. *European Journal of Operational Research, 247*(1), 1–15. https://doi.org/10.1016/j.ejor.2015.05.015

Fazli-Khalaf, M., Khalilpourazari, S., & Mohammadi, M. (2019). Mixed robust possibilistic flexible chance constraint optimization model for emergency blood supply chain network design. *Annals of operations research*, *283*(1), 1079-1109.

Feiring, B. R. (1991). Production planning in stochastic demand environments. Mathematical and computer modelling, 15(10), 91-95.

Flexis AG. (n.d.). The benefits of Industry 4.0 in production planning. *Flexis Blog*. Retrieved from https://blog.flexis.com/the-benefits-of-industry-4.0-in-production-planning

Frey, C. B., & Osborne, M. A. (2017). The future of employment: How susceptible are jobs to computerisation?. *Technological forecasting and social change*, *114*, 254-280.

Friedman, J. H. (1991). Multivariate Adaptive Regression Splines. The Annals of Statistics, 19(1), 1–67. https://doi.org/10.1214/aos/1176347963

Galbraith, J. R. (1973). Designing complex organizations. *Reading/Addison Wesley*.

Garre, A., Ruiz, M. C., & Hontoria, E. (2020). Application of Machine Learning to support production planning of a food industry in the context of waste generation under uncertainty. Operations Research Perspectives, 7, 100147.

Gearhart, J. L., Adair, K. L., Durfee, J. D., Jones, K. A., Martin, N., & Detry, R. J. (2013). *Comparison of open-source linear programming solvers* (No. SAND2013-8847). Sandia National Lab.(SNL-NM), Albuquerque, NM (United States).

Gezen, M., & Karaaslan, A. (2022). Energy planning based on Vision-2023 of Turkey with a goal programming under fuzzy multi-objectives. Energy, 261, 124956.

Ghazvini, M. A. F., & Ramezanian, R. (2013). A decision support system for aggregate production planning under uncertainty: A case study. *International Journal of Production Research, 51*(18), 5455–5469. https://doi.org/10.1080/00207543.2013.794320

Gholamian, N., Mahdavi, I., & Tavakkoli-Moghaddam, R. (2016). Multi-objective multi-product multi-site aggregate production planning in a supply chain under uncertainty: fuzzy multi-objective optimisation. International Journal of Computer Integrated Manufacturing, 29(2), 149-165.

Goguen, J. A. (1973). LA Zadeh. Fuzzy sets. Information and control, vol. 8 (1965), 338–353. - Zadeh, L. A. (1971). Similarity relations and fuzzy orderings. Information sciences, vol. 3, 177-200. The Journal of Symbolic Logic, 38(4), 656-657.

Goli, A., Tirkolaee, E. B., Malmir, B., Bian, G. B., & Sangaiah, A. K. (2019). A multi-objective invasive weed optimization algorithm for robust aggregate production planning under uncertain seasonal demand. Computing, 101(6), 499-529.

Gómez-Rocha, J. E., Hernández-Gress, E. S., & Santos-Borbolla, C. A. (2024). Improving aggregate production planning considering maximum inventory area and service level with demand uncertainty: a nearshoring context in Mexican companies. Journal of Industrial and Production Engineering, 1-14.

González Rodríguez, G., Gonzalez-Cava, J. M., & Méndez Pérez, J. A. (2020). An intelligent decision support system for production planning based on machine learning. Journal of Intelligent Manufacturing, 31(5), 1257-1273.

Gopalakrishnan, K., Xu, X., & Deb, K. (2021). Human paradigm and reliability for aggregate production planning under uncertainty. International Journal of Production Research, 60(2), 289-310. Retrieved from https://www.researchgate.net/publication/361664852

Graczyk-Kucharska, M., Olszewski, R., & Weber, G.-W. (2023). The use of spatial data mining methods for modeling HR Challenges of Generation Z in Greater Poland Region. Central European Journal of Operations Research (CEJOR), 31, 205-237. https://doi.org/10.1007/s10100-022-00805-5

Graczyk-Kucharska, M., Özmen, A., Szafrański, M., Weber, G.- W., Goliński, M., & Spychała, M. (2020). Knowledge accelerator by transversal competences and multivariate adaptive regression splines. CEJOR, 28(2): 645–669. https://doi.org/10.1007/s10100-019-00636-x

Graczyk-Kucharska, M., Szafrański, M., Gütmen, S., Goliński, M., Spychała, M., Weber, G.-W., Włodarczak, Z., Kuter, S., & Özmen, A. (2020). Modeling for Human Resources Management by Data Mining, Analytics and Artificial Intelligence in the Logistics Departments. In: P. Golinska-Dawson, K.-M. Tsai, & M. Kosacka-Olejnik (Eds.), Smart and

Sustainable Supply Chain and Logistics – Trends, Challenges, Methods and Best Practices (pp. 291–303). Springer International Publishing. https://doi.org/10.1007/978-3-030-61947-3_20

Graczyk-Kucharska, M., Weber, G.-W., Szafranski, M. Stachurski T., & Wozna, D. (2022). Model dopasowania kandydata do oferty z wykorzystaniem metody MARSplines i sieci neuronowych. In: *Zarzadzanie Kompetencjami - Wspomagane Metodami Statystcznymi*, M. Szafranski, M. Golinski, M. Graczyk-Kucharska and M. Spychala, Wydawnictwo Politechniki Poznanskiej, Poznan.

Graves, S. C. (1999). Manufacturing planning and control. In: S. C. Graves, A. H. Rinnooy Kan, & P. H. Zipkin (Eds.), *Logistics of Production and Inventory*: 729–761. Amsterdam: *Elsevier*.

Güçdemir, H., & Selim, H. (2017). Customer centric production planning and control in job shops: A simulation optimization approach. Journal of Manufacturing Systems, 43, 100-116.

Gyulai, D., Pfeiffer, A., Kádár, B., & Monostori, L. (2014). Capacity analysis and planning for flexible assembly lines. In International Automation Congress 2014, 1-9.

Haarhaus, B. (2018). Uncovering cognitive and affective sources of satisfaction homogeneity in work teams. Group Processes & Intergroup Relations, 21(4), 646–668. https://doi.org/10.1177/1368430216684542

Hahn, G. J., & Brandenburg, M. (2018). A sustainable aggregate production planning model for the chemical process industry. Computers & operations research, 94, 154-168.

Hashemi Doulabi, H., & Khalilpourazari, S. (2023). Stochastic weekly operating room planning with an exponential number of scenarios. Annals of Operations Research, 328(1), 643-664.

Hax, A. C., & Meal, H. C. (1975). Hierarchical integration of production planning and scheduling. In M. A. Geisler (Ed.), *Studies in Management Sciences* (pp. 53–69). Amsterdam: Elsevier.

Hecklau, F., Galeitzke, M., Flachs, S., & Kohl, H. (2016). Holistic approach for human resource management in Industry 4.0. *Procedia cirp*, *54*, 1-6.

Heizer, J., Render, B., & Munson, C. (2020). *Operations management: Sustainability and supply chain management* (13th ed.). Upper Saddle River, NJ: Pearson Education.

Herrbach, O., & Mignonac, K. (2004). How organisational image affects employee attitudes. Human Resource Management Journal, 14(4), 76–88. https://doi.org/10.1111/j.1748-8583.2004.tb00134.x

Hespanha, J. P. (2004, March). Stochastic hybrid systems: Application to communication networks. In *International Workshop on Hybrid Systems: Computation and Control* (pp. 387-

401). Berlin, Heidelberg: Springer Berlin Heidelberg. Hirsch-Kreinsen, H. (2016). Digitization of industrial work: Development paths and prospects. Journal for Labour Market Research, 49(1), 1–14. https://doi.org/10.1007/s12651-016-0200-6

Ho, C. J. (1989). Evaluating the impact of operating environments on MRP system nervousness. The International Journal of Production Research, 27(7), 1115-1135.

Holt, C. C., Modigliani, F., & Simon, H. A. (1955). A linear decision rule for production and employment scheduling. *Management Science*, *2*(1), 1-30.

Holt, C. C., Modigliani, F., Muth, J. F., & Simon, H. A. (1960). Planning production, inventories, and workforce. *Prentice Hall*.

Islam, S. R., Novoa, C., & Jin, T. (2022). Multi-facility aggregate production planning with prosumer microgrid: A two-stage stochastic program. Journal of Cleaner Production, 367, 132911.

Ivanov, D., & Dolgui, A. (2020). A digital supply chain twin for managing the disruption risks and resilience in the era of Industry 4.0. Transportation Research Part E, 136, 101933.

Ivanov, D., & Dolgui, A. (2020). Viable supply chain model: Integrating agility, resilience, and sustainability perspectives—Lessons from COVID-19. International Journal of Production Research, 59(10), 1-18.

Ivanov, D., Dolgui, A., Sokolov, B., Werner, F., & Ivanova, M. (2023). Digital twins for production planning: A review and future perspectives. Journal of Manufacturing Systems, 64, 117-134. Retrieved from https://www.tandfonline.com/doi/full/10.1080/21693277.2023.2279329

Jabrouni, H., Jaber, M., & El-Hajjar, M. (2022). Workforce planning in production: A human-centric perspective. Journal of Industrial Engineering and Management, 15(4), 513-528. Retrieved from https://www.jiem.org/index.php/jiem/article/download/451/297

Jäckel, P. (2002). Monte Carlo Methods in Finance. *John Wiley & Sons*.

James, G., Witten, D., Hastie, T., Tibshirani, R., & Taylor, J. (2023). Statistical Learning. In G. James, D. Witten, T. Hastie, R. Tibshirani, & J. Taylor (Eds.), An Introduction to Statistical Learning: With Applications in Python: 15–67. *Springer International Publishing*. https://doi.org/10.1007/978-3-031-38747-0_2

Jamalnia, A., Yang, J.-B., Xu, D.-L., & Feili, A. (2017). Novel decision model based on mixed chase and level strategy for aggregate production planning under uncertainty: Case study in beverage industry. Computers & Industrial Engineering, 114, 54-68.

Jamalnia, A., Yang, J. B., Feili, A., Xu, D. L., & Jamali, G. (2019). Aggregate production planning under uncertainty: a comprehensive literature survey and future research directions. The International Journal of Advanced Manufacturing Technology, 102(1), 159-181.

Jang, J., & Do Chung, B. (2020). Aggregate production planning considering implementation error: A robust optimization approach using bi-level particle swarm optimization. Computers & Industrial Engineering, 142, 106367.

Jiménez, M., Arenas, M., Bilbao, A., & Rodrı, M. V. (2007). Linear programming with fuzzy parameters: an interactive method resolution. European journal of operational research, 177(3), 1599-1609.

Judge, T. A., & Watanabe, S. (1993). Another look at the job satisfaction-life satisfaction relationship. Journal of Applied Psychology, 78(6), 939.

Jung, Y., & Suh, Y. (2019). Mining the voice of employees: A text mining approach to identifying and analyzing job satisfaction factors from online employee reviews. Decision Support Systems, 123, 113074. https://doi.org/10.1016/j.dss.2019.113074

Kagermann, H., Wahlster, W., & Helbig, J. (2013). Securing the future of German manufacturing industry: Recommendations for implementing the strategic initiative Industry 4.0**.** Final report of the Industrie 4.0 working group.

Kaufman, A., & Gupta, M. M. (1991). *Introduction to fuzzy arithmetic*. New York: *Van Nostrand Reinhold Company*.

Khalilpourazari, S., Mirzazadeh, A., Weber, G. W., & Pasandideh, S. H. R. (2020b). A robust fuzzy approach for constrained multi-product economic production quantity with imperfect items and rework process. Optimization. https://doi.org/10.1080/02331934.2019.1630625

Khalili-Damghani, K., & Shahrokh, A. (2014). Solving a new multi-period multi-objective multi-product aggregate production planning problem using fuzzy goal programming. Industrial Engineering and Management Systems, 13(4), 369-382.

Khalilpourazari, S., Soltanzadeh, S., Weber, G. W., & Roy, S. K. (2020a). Designing an efficient blood supply chain network in crisis: neural learning, optimization and case study. Annals of Operations Research, 289, 123-152.

Khalilpourazari, S., Weber, G.-W., & Pasandideh, S.H.R. (2020). A robust fuzzy approach for constrained multi-product economic production quantity with imperfect items and rework process, Optimization 69(1), 63-90. https://doi.org/10.1080/02331934.2019.1630625

Khalilpourazari, S., & Hashemi Doulabi, H. (2023). A flexible robust model for blood supply chain network design problem. *Annals of operations research*, *328*(1), 701-726.

Khansa, L., Zobel, C. W., & Goicochea, G. (2012). Creating a taxonomy for mobile commerce innovations using social network and cluster analyses. *International Journal of Electronic Commerce*, *16*(4), 19-52.

King, R. E., & Love, C. E. (1980). Aggregate production planning with stochastic demand. Journal of Operations Management, 1(1), 43-52. https://doi.org/10.1016/0272-6963(80)90023-7

Krajewski, L. J., & Ritzman, L. P. (1996). Operations management: Strategy and analysis. *Addison-Wesley*.

Krishnan, R., Phan, P. Y., Krishnan, S. N., Agarwal, R., & Sohal, A. (2024). Industry 4.0-driven business model innovation for supply chain sustainability: An exploratory case study. Business Strategy and the Environment. https://doi.org/10.1002/bse.3970

Kuchta, D. (2010). The use of fuzzy numbers in practical project planning and control. In: *Fuzzy Optimization: Recent Advances and Applications*. Springer: 323-339.

Kuvvetli, Y. (2023). A goal programming model for two-stage COVID19 test sampling centers location-allocation problem. Central European Journal of Operations Research, 31(1), 1-20.

Lasdon, L. S., & Terjung, R. C. (1971). An efficient algorithm for multi-item scheduling. *Operations research*, *19*(4), 946-969.

Lasi, H., Fettke, P., Kemper, H. G., Feld, T., & Hoffmann, M. (2014). Industry 4.0. Business & information systems engineering, 6, 239-242.

Lee, J., Bagheri, B., & Kao, H. A. (2015). A cyber-physical systems architecture for Industry 4.0-based manufacturing systems. *Manufacturing Letters, 3*(7), 18-23. https://doi.org/10.1016/j.mfglet.2014.12.001.

Lee, Y. Y. (1990). *Fuzzy sets theory approach to aggregate production planning and inventory control*. Kansas State University.

Li, D. F., Nan, J. X., & Zhang, M. J. (2010). A ranking method of triangular intuitionistic fuzzy numbers and application to decision making. International Journal of Computational Intelligence Systems, 3(5), 522-530.

Liao, P., Jiang, L., Liu, B., Chen, C., Fang, D., Rao, P., & Zhang, M. (2014). A cognitive perspective on the safety communication factors that affect worker behavior. Journal of Building Construction and Planning Research, 2(03), 183.

Liao, Y., Deschamps, F., Loures, E. D. F. R., & Ramos, L. F. P. (2017). Past, present and future of Industry 4.0—A systematic literature review and research agenda proposal. International Journal of Production Research, 55(12), 3609–3629. https://doi.org/10.1080/00207543.2017.1308576

Lin, B., Bouneffouf, D., & Cecchi, G. (2022). Predicting human decision making in psychological tasks with recurrent neural networks. PLOS ONE, 17(5), e0267907. https://doi.org/10.1371/journal.pone.0267907

Liu, L. F., & Yang, X. F. (2021). Multi-objective aggregate production planning for multiple products: A local search-based genetic algorithm optimization approach. International Journal of Computational Intelligence Systems, 14(1), 1-16.

Luo, J., Thevenin, S., & Dolgui, A. (2022). Industry 4.0 technologies for production planning and control: A systematic review and future research directions. International Journal of Production Research, 60(4), 1024-1043. Retrieved from https://hal.science/hal-03793675

Madadi, N., & Wong, K. Y. (2014). A multiobjective fuzzy aggregate production planning model considering real capacity and quality of products. Mathematical problems in engineering, 2014(1), 313829.

Maity, G., & Roy, S. K. (2017). Solving fuzzy transportation problem using multi-choice goal programming. Discrete Mathematics, Algorithms and Applications, 9(06), 1750076.

Manandhar, S., Tarim, A., & Walsh, T. (2009). Scenario-based stochastic constraint programming. *arXiv preprint arXiv:0905.3763*.

Manne, A. S. (1960). Linear programming and sequential decisions. *Management Science, 6*(3), 259-267. https://doi.org/10.1287/mnsc.6.3.259

Medrano, L. A., & Trógolo, M. A. (2018). Employee well-being and life satisfaction in Argentina: The contribution of psychological detachment from work. Revista de Psicología Del Trabajo y de Las Organizaciones, 34(2), 69-81.

Mehdizadeh, E., Niaki, S. T. A., & Hemati, M. (2018). A bi-objective aggregate production planning problem with learning effect and machine deterioration: Modeling and solution. Computers & Operations Research, 91, 21-36.

Mirzapour Al-E-Hashem, S. M. J., Malekly, H., & Aryanezhad, M. B. (2011). A multi-objective robust optimization model for multi-product multi-site aggregate production planning in a supply chain under uncertainty. International Journal of Production Economics, 134(1), 28-42.

Mirzapour Al-e-hashem, S. M. J., Baboli, A., & Sazvar, Z. (2013). A stochastic aggregate production planning model in a green supply chain: Considering flexible lead times, nonlinear purchase and shortage cost functions. European Journal of Operational Research, 230(1), 26-41.

Mittelmann, H. D. (2002). Benchmark for optimization software. *http://plato. asu. edu/bench. html*.

Modarres, M., & Izadpanahi, E. (2016). Aggregate production planning by focusing on energy saving: A robust optimization approach. Journal of Cleaner Production, 133, 1074-1085.

Monostori, L., & Kovács, G. (2023). Human-centricity in the design of production planning and control systems: A first approach towards Industry 5.0. *Procedia CIRP, 112*, 188-193. Retrieved from https://www.researchgate.net/publication/364795447

Moosavi, J., Bakhshi, J., & Martek, I. (2021). The application of industry 4.0 technologies in pandemic management: Literature review and case study. Healthcare Analytics, 1, 100008.

Morariu, C., Morariu, O., Răileanu, S., & Borangiu, T. (2020). Machine learning for predictive scheduling and resource allocation in large scale manufacturing systems. Computers in Industry, 120, 103244.

Mori, J., & Mahalec, V. (2015). Planning and scheduling of steel plates production. Part I: Estimation of production times via hybrid Bayesian networks for large domain of discrete variables. Computers & Chemical Engineering, 79, 113-134.

Mula, J., Poler, R., & García-Sabater, J. P. (2006). Models for production planning under uncertainty: A review. International Journal of Production Economics, 103(1), 271–285. https://doi.org/10.1016/j.ijpe.2005.09.001

Nam, S. J., & Logendran, R. (1992). Aggregate production planning—a survey of models and methodologies. European Journal of Operational Research, 61(3), 255-272.

Netlib. Testcases for real world LP instances. Software, 2012. URL ftp://ftp.netlib. org/lp/data/index.html.

Neumann, W. P., Kolus, A., & Wells, R. W. (2016). Human factors in production system design and quality performance–a systematic review. Ifac-Papersonline, 49(12), 1721–1724.

Newman, W. R., & Yano, C. A. (1979). Multi-item production planning under uncertainty. *Management Science, 25*(3), 259-274. https://doi.org/10.1287/mnsc.25.3.259

Nguyen, H. T. (1978). A note on the extension principle for fuzzy sets. Journal of mathematical analysis and applications, 64(2), 369-380.

Nicolescu, G., & Mosterman, P. J. (2018). *Model-based design for embedded systems*. Crc Press.

Ning, Y., Liu, J., & Yan, L. (2013). Uncertain aggregate production planning. Soft Computing, 17, 617-624.

Noegraheni, E., & Nuradli, H. (2016). Aggregate planning to minimize cost of production in manufacturing company. Binus Business Review, 7(1), 39-45.

Nugraha, I., Hisjam, M., & Sutopo, W. (2020, October). Aggregate planning method as production quantity planning and control to minimizing cost. In IOP Conference Series: Materials Science and Engineering (Vol. 943, No. 1, p. 012045). IOP Publishing.

O'Hora, D., & Maglieri, K. A. (2006). Goal Statements and Goal-Directed Behavior: A Relational Frame Account of Goal Setting in Organizations. Journal of Organizational Behavior Management, 26(1–2), 131–170. https://doi.org/10.1300/J075v26n01_06

Özmen, A., Batmaz, I., & Weber, G.-W. (2014). Precipitation Modeling by Polyhedral RCMARS and Comparison with MARS and CMARS. Environmental Modeling and Assessment: 19 (82), 425-435.

Özmen, A., Kropat, E., & Weber, G.-W. (2014). Spline regression models for complex multi-modal regulatory networks. Optimization Methods and Software (OMS), 29(3), 515-534.

Özmen, A., & Weber G.-W. (2012). Robust conic generalized partial linear models using RCMARS method. AIP Proceedings 1499, 337-343. (ISBN 978-0-7354-1113-5, ISSN 0094-243X).

Özmen, A., & Weber, G.-W. (2014). RMARS: Robustification of Multivariate Adaptive Regression Spline under Polyhedral Uncertainty. Journal of Computational and Applied Mathematics (CAM), 259, 914-924.

Özmen, A., Weber, G.-W., Çavuşoğlu, Z., & Defterli, Ö. (2013). The new robust conic GPLM method with an Application to Finance: prediction of credit default, Journal of Global Optimization (JOGO), 56(2), 233-249.

Özmen, A., Zinchenko, Y., & Weber, G.-W. (2023). Robust multivariate adaptive regression splines under cross-polytope uncertainty - an application in natural gas market, ANOR, 324, 1337–1367. https://doi.org/10.1007/s10479-022-04993-w

Pérez-Lechuga, G., Venegas-Martínez, F., & Martínez-Sánchez, J. F. (2021). Mathematical modeling of manufacturing lines with distribution by process: A markov chain approach. Mathematics, 9(24), 3269.

Pfeiffer, S. (2016). Robots, Industry 4.0 and humans, or why assembly work is more than routine work. Socio-Economic Review, 14(2), 239–261. https://doi.org/10.1093/ser/mwv03

PlanetTogether. (n.d.). Human-centric approaches to production scheduling in medical manufacturing. Retrieved from https://www.planettogether.com/blog/human-centric-approaches-to-production-scheduling-in-medical-manufacturing

Rahmani, D., Zandi, A., Behdad, S., & Entezaminia, A. (2021). A light robust model for aggregate production planning with consideration of environmental impacts of machines. Operational Research, 21(1), 273-297.

Ramezanian, R., Rahmani, D., & Barzinpour, F. (2012). An aggregate production planning model for two phase production systems: Solving with genetic algorithm and tabu search. Expert Systems with Applications, 39(1), 1256–1263.

Rasmi, S. A. B., Kazan, C., & Türkay, M. (2019). A multi-criteria decision analysis to include environmental, social, and cultural issues in the sustainable aggregate production plans. Computers & Industrial Engineering, 132, 348-360.

Rossi, F., Van Beek, P., & Walsh, T. (2008). Constraint programming. *Foundations of Artificial Intelligence*, *3*, 181-211.

Roy, S. K., Maity, G., & Weber, G. W. (2017). Multi-objective two-stage grey transportation problem using utility function with goals. Central European Journal of Operations Research, 25, 417-439.

Rüßmann, M., Lorenz, M., Gerbert, P., Waldner, M., Justus, J., Engel, P., & Harnisch, M. (2015). Industry 4.0: The future of productivity and growth in manufacturing industries**.** *Boston Consulting Group*.

Said, H. M., Abukraa, M. K., & Rose, R. M. (2015). The relationship between personality and job satisfaction among employees in Libyan oil and gas company. American Research Journal of Business and Management, 1(2), 18–24.

Savage, S., Scholtes, S., & Zweidler, D. (2006). Probability management: planning for an uncertain future calls for a shift in information management--from single numbers to probability distributions--in order to correct the" flaw of averages." This, in turn, gives rise to the prospect of a Chief Probability Officer to manage the distributions that underlie risk, real portfolios, real options and many other activities in the global economy. OR/MS Today, 33(1), 20-28.

Schwab, K. (2017). *The fourth industrial revolution*. Crown Currency.

Sethi, S. P., Yan, H., Zhang, H., & Zhang, Q. (2002). Optimal and hierarchical controls in dynamic stochastic manufacturing systems: A survey. *Manufacturing & Service Operations Management*, *4*(2), 133-170.

Sgarbossa, F., Grosse, E. H., Neumann, W. P., Battini, D., & Glock, C. H. (2020). Human factors in production and logistics systems of the future. Annual Reviews in Control, 49, 295-305. https://doi.org/10.1016/j.arcontrol.2020.04.007

Shapiro, J. F., “Mathematical Programming Models and Methods for Production Planning and Scheduling,” In Handbooks in Operations Research and Management Science, Volume 4, Logistics of Production and Inventory, edited by S. C. Graves, A. H. G. Rinnooy Kan and P. H. Zipkin, Amsterdam, Elsevier Science Publishers B. V., 1993, 371-443.

Shapiro, A., Dentcheva, D., & Ruszczynski, A. (2014). *Lectures on stochastic programming: Modeling and theory* (2nd ed.). SIAM. https://doi.org/10.1137/1.9781611973433

Shi, X., & Xu, S. (2016). A hybrid model for reliability analysis in uncertain environments. *Reliability Engineering & System Safety*, 152, 131–141.

Silver, E. A., Pyke, D. F., & Thomas, D. (1998). *Inventory and Production Management in Supply Chains* (3rd ed.). *CRC Press*.

Sima, V., Gheorghe, I. G., Subić, J., & Nancu, D. (2020). Influences of the industry 4.0 revolution on the human capital development and consumer behavior: A systematic review. Sustainability, 12(10), 4035.

Steel, P., Schmidt, J., Bosco, F., & Uggerslev, K. (2019). The effects of personality on job satisfaction and life satisfaction: A meta-analytic investigation accounting for bandwidth–fidelity and commensurability. Human Relations, 72(2), 217–247. https://doi.org/10.1177/0018726718771465

Stevenson, W. J. (2011). *Operations management* (11th ed.). McGraw-Hill Education.

Szafrański, M., Gütmen, S., Graczyk-Kucharska, M., & Weber, G. W. (2022). Modelling IT Specialists Competency in the Era of Industry 4.0. In: J. Machado, F. Soares, J. Trojanowska, & S. Yildirim (Eds.), Innovations in Mechatronics Engineering, 257-269. *Springer International Publishing*. https://doi.org/10.1007/978-3-030-79168-1_24

Tang, J., Fung, R. Y., & Yung, K. L. (2003). Fuzzy modelling and simulation for aggregate production planning. International Journal of Systems Science, 34(12-13), 661-673.

Taylan, P., Weber G.-W., & Yerlikaya, F. (2010). A new approach to multivariate adaptive regression spline by using Tikhonov regularization and continuous optimization. TOP, 18(2), 377-395.

Taylan, P., & Weber, G. W. (2019). CG-lasso estimator for multivariate adaptive regression spline. Mathematical Methods in Engineering: Applications in Dynamics of Complex Systems, 121-136.

Thomas. L. J., & McClain, J. O. “An Overview of Production Planning,” In Handbooks in Operations Research and Management Science, Volume 4, Logistics of Production and Inventory, edited by S. C. Graves, A. H. G. Rinnooy Kan and P. H. Zipkin, Amsterdam, *Elsevier Science Publishers B. V.*, 1993, pp. 333-370.

Tirkolaee, E. B., Goli, A., & Weber, G. W. (2019). Multi-objective aggregate production planning model considering overtime and outsourcing options under fuzzy seasonal demand. In International Scientific-Technical Conference Manufacturing, 81-96. *Springer*, Cham.

Tirkolaee, E. B., Aydin, N. S., & Mahdavi, I. (2022). A hybrid biobjective markov chain based optimization model for sustainable aggregate production planning. IEEE Transactions on Engineering Management, 71, 4273-4283.

Tirkolaee, E. B., Aydin, N. S., & Mahdavi, I. (2023). A bi-level decision-making system to optimize a robust-resilient-sustainable aggregate production planning problem. Expert Systems with Applications, 228, 120476.

Tortorella, G. L., Giglio, R., & van Dun, D. H. (2019). Industry 4.0 adoption as a moderator of the impact of lean production practices on operational performance improvement. International Journal of Operations & Production Management, 39(6), 860-886. https://doi.org/10.1108/IJOPM-01-2019-0005

Tu, Y., Lu, X., & Yu, Y. (2017). Supervisors' Ethical Leadership and Employee Job Satisfaction: A Social Cognitive Perspective. Journal of Happiness Studies, 18(1), 229-245.

Türkay, M., Saraçoğlu, Ö., & Arslan, M. C. (2016). Sustainability in supply chain management: Aggregate planning from sustainability perspective. PloS one, 11(1), e0147502.

Uğurlu, K. (2017). Controlled Markov decision processes with AVaR criteria for unbounded costs. Journal of Computational and Applied Mathematics, 319, 24-37.

Uğurlu, K. (2018). Robust optimal control using conditional risk mappings in infinite horizon. Journal of Computational and Applied Mathematics, 344, 275-287.

Van Der Schaft, A. J., & Schumacher, H. (2007). *An introduction to hybrid dynamical systems* (Vol. 251). *Springer*.

Wang, K., & Zhang, Y. (2021). Applications of machine learning in smart manufacturing: A review. IEEE Transactions on Industrial Informatics, 17(12), 8414-8433. Retrieved from https://arxiv.org/abs/2106.12916

Wang, P. P., & Fang, S. C. (1988). Aggregate production planning with multiple objectives in a fuzzy environment. Fuzzy Sets and Systems, 26(1), 31-42. https://doi.org/10.1016/0165-0114(88)90076-4

Wang, S., Wan, J., Li, D., & Zhang, C. (2016). Implementing smart factory of Industry 4.0: An outlook. International Journal of Distributed Sensor Networks, 12(1), 3159805. https://doi.org/10.1177/1550147715629701

Waschneck, B., Reichstaller, A., Belzner, L., Altenmüller, T., Bauernhansl, T., Knapp, A., & Kyek, A. (2018). Optimization of global production scheduling with deep reinforcement learning. Procedia Cirp, 72, 1264-1269.

Weber, G.-W., Batmaz, I., Köksal, G., Taylan, P., & Yerlikaya-Özkurt, F. (2012). CMARS: A new contribution to nonparametric regression with multivariate adaptive regression splines supported by continuous optimisation. Inverse Problems in Science and Engineering (IPSE): 20(3), 371-400.

Weiss, H. M. (2002). Deconstructing job satisfaction: Separating evaluations, beliefs and affective experiences. Human Resource Management Review, 12(2), 173–194. https://doi.org/10.1016/S1053-4822(02)00045-1

Więcek-Janka, E. (2016). THE DIRECTION OF THE ALLOCATION OF PROFIT AS A DETERMINANT OF THE BUSINESS OF FAMILY RESPONSIBILITY (RESEARCH RESULTS), 75-93.

Wu, T., Huang, L., Liang, Z., Zhang, X., & Zhang, C. (2021). A supervised learning-driven heuristic for solving the facility location and production planning problem. European Journal of Operational Research, 301(2), 785-796.

Xu, L. D., Xu, E. L., & Li, L. (2018). Industry 4.0: State of the art and future trends. International Journal of Production Research, 56(8), 2941–2962. https://doi.org/10.1080/00207543.2018.1444806

Xu, M., David, J. M., & Kim, S. H. (2021). The fourth industrial revolution: Opportunities and challenges. International Journal of Financial Research, 12(2), 90-100. https://doi.org/10.5430/ijfr.v12n2p90

Yano, C. A., & Lee, H. L. (1995). Lot sizing with random yields: A review. Operations research, 43(2), 311-334.

Yin, K. K., Liu, H., & Yin, G. G. (2003). Stochastic models and numerical solutions for production planning with applications to the paper industry. Computers & Chemical Engineering, 27(11), 1693-1706.

Yu, C., Zhang, W., Xu, X., Ji, Y., & Yu, S. (2018). Data mining based multi-level aggregate service planning for cloud manufacturing. Journal of Intelligent Manufacturing, 29(6), 1351-1361.

Yu, G. F., & Li, D. F. (2022). A novel intuitionistic fuzzy goal programming method for heterogeneous MADM with application to regional green manufacturing level evaluation under multi-source.

Zadeh, L. A. (1965). Fuzzy sets. Information and Control. 8(3), 338-353.

Zhang, F., Ignatius, J., Lim, C. P., & Zhao, Y. (2014). A new method for ranking fuzzy numbers and its application to group decision making. *Applied mathematical modelling*, *38*(4), 1563-1582.

Zimmermann, H. J. (1991). *Fuzzy set theory - and its applications*. *Kluwer Academic Publishers*. https://doi.org/10.1007/978-94-015-7949-1

Zimmermann, H. J. (2011). *Fuzzy set theory - and its applications*. *Springer Science & Business Media*.